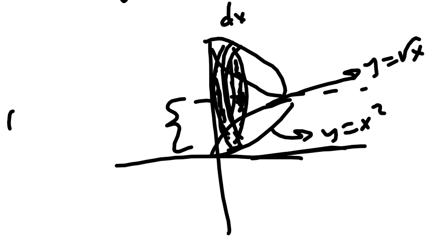
$y=x^2$, $x=y^2$, about $y=1$

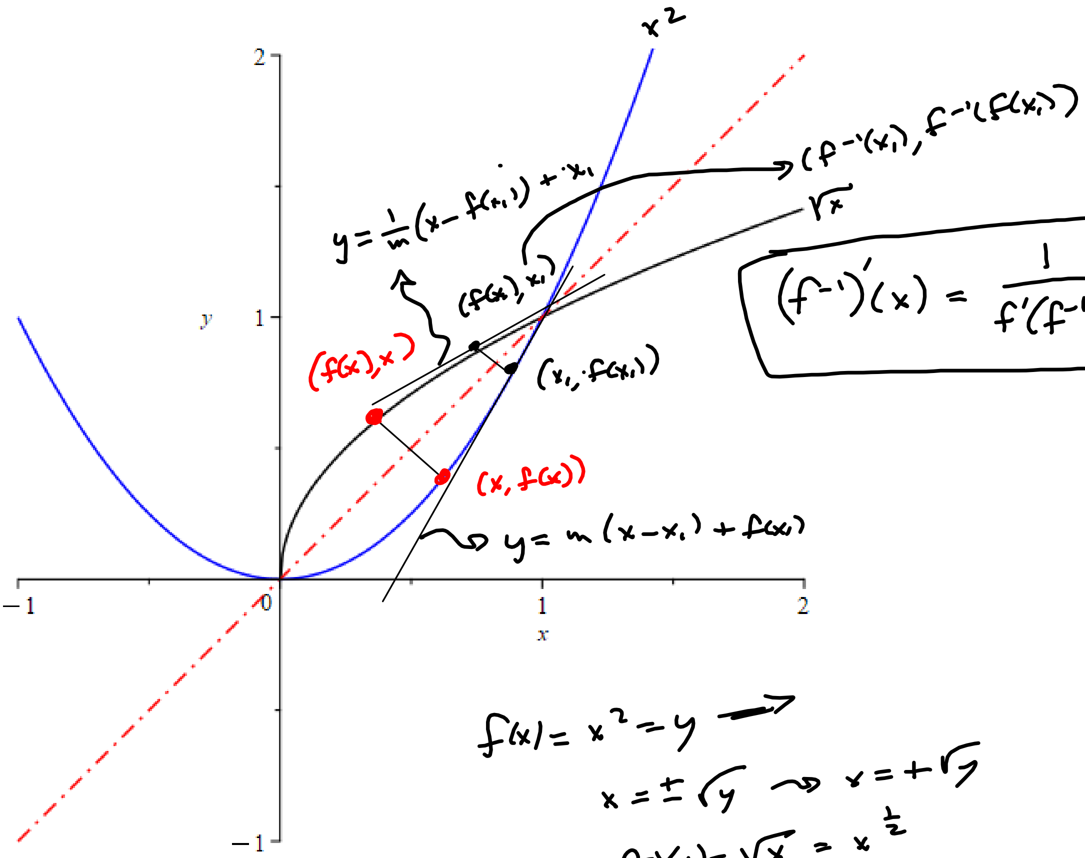


$$\pi\int_0^1 (\text{OUTER}^2-\text{INNER}^2)$$

$$=\pi\int_0^1 \left((1-x^2)^2-(1-\sqrt{x})^2\right)dx$$

$$x=y^2 \Rightarrow y=\pm\sqrt{x} \rightsquigarrow +\sqrt{x}$$

§6.1 Inverse Functions & their Derivatives



$$\boxed{(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}}$$

$f(x) = x^2 = y \longrightarrow$

$x = \pm\sqrt{y} \rightsquigarrow x = +\sqrt{y}$

$f^{-1}(x) = \sqrt{x} = x^{\frac{1}{2}}$

$(f^{-1})'(x) = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$ Directly

$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))} =$

$f(x) = x^2$

$f'(x) = 2x$

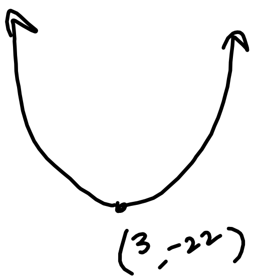
$\frac{1}{f'(f^{-1}(x))} = \frac{1}{2(\sqrt{x})}$

Why do we care about this formula?

Sometimes $f^{-1}(x)$ is hard to compute, but finding f' & $f^{-1}(x_0)$ for a single x_0 is not that hard.

$f(x) = x^2 - 6x - 13 = (x^2 - 6x + 3^2) - 9 - 13$

$= (x-3)^2 - 22 = y$



on $(-\infty, 3]$, $f^{-1}(x) = 3 - \sqrt{x+22}$

on $[3, \infty)$, $f^{-1}(x) = 3 + \sqrt{x+22}$

Switch x & y. Solve for y:

$(y-3)^2 - 22 = x$

$(y-3)^2 = x + 22$

$y - 3 = \pm\sqrt{x+22}$

$y = 3 \pm \sqrt{x+22} \rightsquigarrow y = 3 + \sqrt{x+22}$

if we use $x \geq 3$ for $\mathcal{D}(f)$

If we take $x \leq 3$ " $\mathcal{D}(f)$,

then $f^{-1}(x) = 3 - \sqrt{x+22}$.

Finding $f^{-1}(x)$ can be painful!

$f(x) = 3x^3 + 4x^2 + 6x + 5$, $a = 5$

Find $(f^{-1})'(a)$

Find $f^{-1}(x)$, compute $(f^{-1})'(x)$, plug in $(f^{-1})'(a)$

Not in the cards, maybe.

But $\underbrace{f^{-1}(5) = 0}_{?}$ is easy!

i.e. Solve $f(x) = 5 \longrightarrow$

$f^{-1}(f(x)) = x = f^{-1}(5)$

$3x^3 + 4x^2 + 6x + 5 = 5$

& $f(0) = 5$ by inspection!

So, All we need is $f'(0)$!

$$(f^{-1})'(5) = \frac{1}{f'(f^{-1}(5))}$$

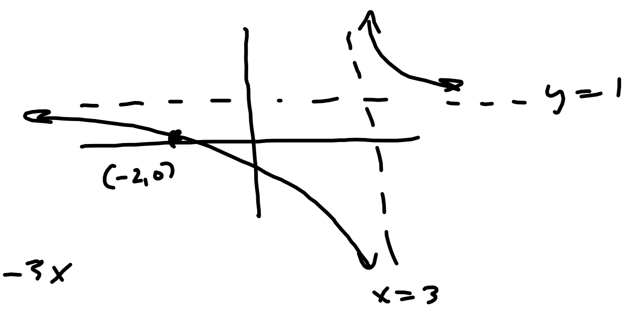
$f'(x) = 9x^2 + 8x + 6$

$\Longrightarrow f'(0) = 6 \longrightarrow \boxed{(f^{-1})'(5) = \frac{1}{6}}$

$f(x) = \frac{x+2}{x-3}$

Find $f^{-1}(x)$

$\frac{y+2}{y-3} = x$ →



$y+2 = x(y-3) = xy - 3x$

$y - xy = -3x - 2$

$y(1-x) = -3x-2$

$y = \frac{-3x-2}{1-x} = \boxed{\frac{3x+2}{x-1} = f^{-1}(x)}$

$y = f(x) = \sqrt{x+7} - 3$

$\sqrt{y+7} - 3 = x$

$\sqrt{y+7} = x+3$

$y+7 = (x+3)^2$

$y = (x+3)^2 - 7 = f^{-1}(x)$