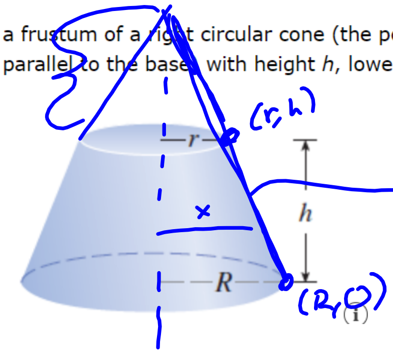
Find the volume V of the described solid S.

a frustum of a right circular cone (the portion of a cone that remains after the tip has been cut off by a plane parallel to the base) with height h, lower base radius R, and top radius r



$(R,0),\ (r,h)$

$$y = \frac{h-0}{r-R}(x-R)+0 \quad = m(x-x_1)+y_1$$

$$= \frac{h}{r-R}(x-R)$$

y-int:

$$\frac{h}{r-R}(-R) = \frac{hR}{R-r}$$

$$V = V_{BIG} - V_{LITTLE}$$

$$= \frac{1}{3}\pi R^2\left(\frac{hR}{R-r}\right) - \frac{1}{3}\pi r^2\left(\frac{hR}{R-r} - \frac{h(R-r)}{R-r}\right)$$

$$= \frac{1}{3}\pi\left(\frac{1}{R-r}\right)\left[R^3h + hr^3\right]$$

$$\boxed{\frac{1}{3}\pi h\left(r^2+rR+R^2\right)}$$

$$y = \frac{h}{r-R}(x-R) \Rightarrow (r-R)y = hx - hR$$

$$\Rightarrow yr - yR + hR = hx \longrightarrow$$

$$x = \frac{yr-yR+hR}{h}$$

$$\pi\int_0^h x^2\,dy \qquad \pi\int_0^h\left(\frac{yr-yR+hR}{h}\right)^2 dy$$

$$= \frac{\pi}{h^2}\int_0^h\left((r-R)y + hR\right)^2 dy$$

$$u = (r-R)y + hR \longrightarrow$$

$$du = (r-R)\,dy$$

$$= \frac{\pi}{(r-R)} \int_0^h \left(\frac{(r-R)y + hR}{h}\right)^2 (r-R)\,dy$$

$$= \frac{\pi}{h^2(r-R)} \left[\frac{((r-R)y + hR)^3}{3}\right]_0^h$$

$$= \frac{\pi}{h^2(r-R)} \left[\frac{((r-R)h + hR)^3}{3}\right]$$

$$= \frac{\pi}{h^2(r-R)} \left[\frac{(rh - Rh + hR)^3}{3}\right] =$$

$y = -x^2+2x \longrightarrow$

$-x^2+2x-y=0$

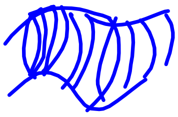
$x^2-2x+y=0$

$x^2-2x \quad = -y$

$x^2-2x+1^2 = -y+1$

$(x-1)^2 = 1-y$

$x = 1 \pm \sqrt{1-y}$



Yesterday's Error.

§5.2:

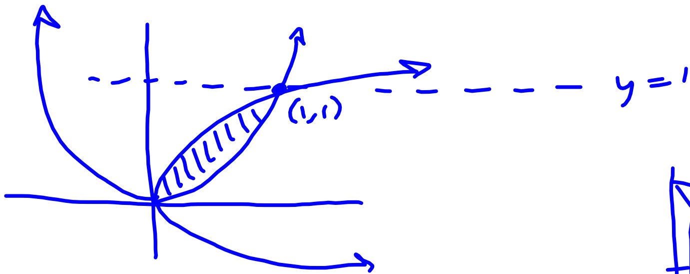
$\pi r^2 h$ = volume of right circular cylinder.

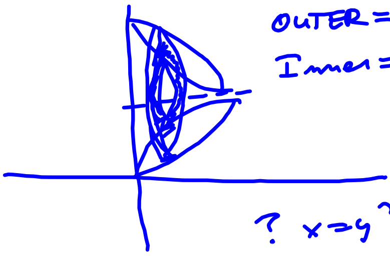
$\pi f(x)^2 dx$

$\pi \int_a^b f(x)^2 dx$ about x-axis

$\pi \int_c^d g(y)^2 dy$ about y-axis.

$y=x^2,\ x=y^2$ about $y=1$





OUTER $= 1 - x^2$

Inner $= 1 - ?$

$= 1 - \sqrt{x}$

$$V = \pi \int_0^1 \left((1-x^2)^2 - (1-\sqrt{x})^2\right) dx$$

$? \ x = y^2 \rightarrow$

$y = \pm\sqrt{x}$

$y = +\sqrt{x}$

$$(x^2-1)^2 - (\sqrt{x}-1)^2 =$$

$$x^4 - 2x^2 + 1 - (x - 2\sqrt{x} + 1)$$

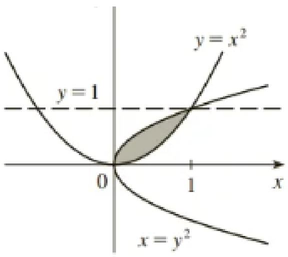
$$= x^4 - 2x^2 + 1 - x - 2\sqrt{x} - 1$$

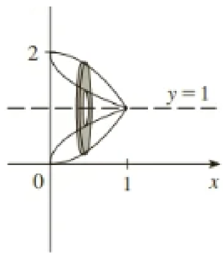
$$= x^4 - 2x^2 - x - 2\sqrt{x}$$

$$\Rightarrow V = \pi \int_0^1 \left(x^4 - 2x^2 - x - 2x^{\frac{1}{2}}\right) dx = \pi \left[\frac{x^5}{5} - \frac{2x^3}{3} - \frac{x^2}{2} - \frac{4}{3}x^{\frac{3}{2}}\right]_0^1$$

$$= \left(\frac{1}{5} - \frac{2}{3} - \frac{1}{2} + \frac{4}{3}\right)\pi = \left(\frac{1}{5} + \frac{2}{3} - \frac{1}{2}\right)\pi = \left(\frac{6 + 20 - 15}{30}\right)\pi = \frac{11}{30}\pi$$

Much better picture.





(a,b), (c,d)

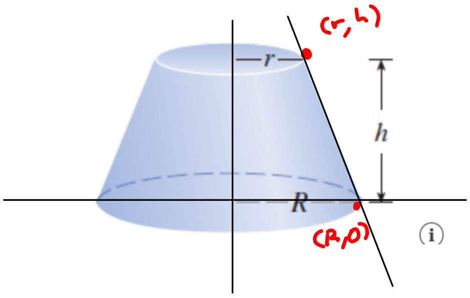
$$y = \frac{d-b}{c-a}(x-a)+b \quad = m(x-x_1)+y_1$$

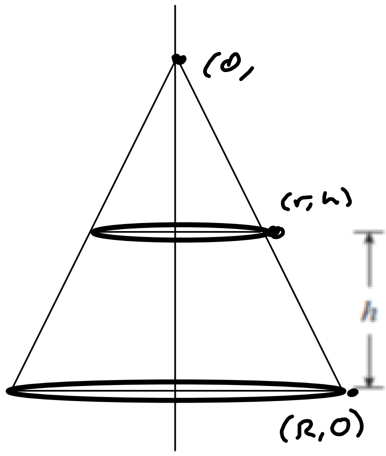
OR

$$\frac{d-b}{c-a}(x-c)+d$$

Find the volume V of the described solid S.

a frustum of a right circular cone (the portion of a cone that remains after the tip has been cut off by a plane parallel to the base) with height h, lower base radius R, and top radius r





$y = \frac{h}{r-R}(x - R)$

$y(0) = \frac{-hR}{r-R} = \frac{hR}{R-r}$

$$\frac{1}{3}\pi h\left(r^2 + rR + R^2\right)$$

BIG CONE - CONE @ TOP

$\left(V = \frac{1}{3}\pi R^2\left(\frac{hR}{R-r}\right) - \frac{\pi r^2}{3}\left(\frac{hR}{R-r} - h\right)\right)(R-r)$

$V(R-r) = \frac{1}{3}\pi R^2(hR) - \frac{\pi}{3}r^2\left(hR - h(R-r)\right)$

$= \frac{\pi}{3}\left(hR^3 + hr^3\right)$

$= \frac{1}{3}\pi R^2\left(\frac{hR}{R-r}\right) - \frac{\pi r^2}{3}\left(\frac{hR}{R-r} - h\right)$

$= \frac{\pi}{3}R^2\left(\frac{hR}{R-r}\right) - \frac{\pi}{3}r^2\left(\frac{hR - hR + hr}{R-r}\right)$

$= \frac{\pi}{3}R^2\left(\frac{hR}{R-r}\right) - \frac{\pi}{3}r^2\left(\frac{hr}{R-r}\right)$

$= \frac{\pi}{3(R-r)}\left[hR^3 - hr^3\right]$

$= \frac{\pi h}{3(R-r)}\left[R^3 - r^3\right]$ Difference of cubes!

$= \frac{\pi h}{3(R-r)}\left[(R-r)(R^2 + rR + r^2)\right]$

$= \boxed{\frac{\pi h}{3}\left[R^2 + rR + r^2\right]}$

$$\int_0^h$$

$$y = \frac{h}{R-r}(x - R)$$

$$y(R-r) = hx - hR$$

$$hx = y(R-r) + hR$$

$$x = \frac{R-r}{h}y + \frac{hR}{R-r} = g(y)$$

$$\pi \int_0^h \left(\frac{R-r}{h}y + \frac{hR}{R-r}\right)^2 dy$$

$$u = \frac{R-r}{h}y + \frac{hR}{R-r}$$

$$du = \frac{R-r}{h}\,dy$$

$$= \frac{\pi h}{R-r}\int_0^h \left(\frac{R-r}{h}y + \frac{hR}{R-r}\right)^2\left(\frac{R-r}{h}\right)dy$$

$$= \frac{\pi h}{R-r}\left[\frac{\left(\frac{R-r}{h}y + \frac{hR}{R-r}\right)^3}{3}\right]_0^h$$

$$= \frac{\pi h}{R-r}\left[\frac{\left(\frac{R-r}{h}(h) + \frac{hR}{R-r}\right)^3}{3} - \frac{\left(\frac{hR}{R-r}\right)^3}{3}\right]$$

$$= \frac{\pi h}{3(R-r)}\left[\left(R-r + \frac{hR}{R-r}\right)^3 - \frac{h^3R^3}{(R-r)^3}\right]$$

$$= \frac{\pi h}{3(R-r)}\left[(R-r)^3 + 3(R-r)^2\left(\frac{hR}{R-r}\right) + 3(R-r)\left(\frac{hR}{R-r}\right)^2 + \cancel{\left(\frac{hR}{R-r}\right)^3} - \cancel{\left(\frac{hR}{R-r}\right)^3}\right]$$

$$= \frac{\pi h}{3(R-r)}\left[(R-r)^3 + 3(R-r)hR + 3\frac{h^2R^2}{R-r}\right]$$

$$=$$