Find the area of the region bounded by $y = x^2 - 4x$ & $y = 2x$

$= x(x-4)$

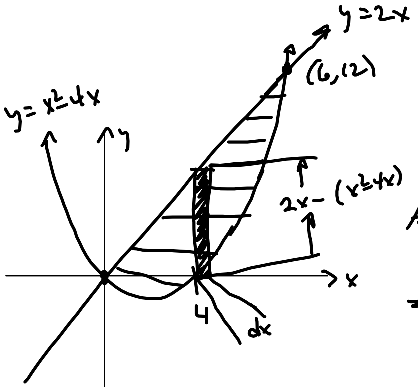
$x^2 - 4x = 2x \longrightarrow$

$x^2 - 6x = 0 \longrightarrow$

$x \in \{0, 6\}$

$2(6) = 12$

$6^2 - 4(6) = 12$



$$\text{Area} = \int_0^6 (\text{upper} - \text{lower})\,dx$$

$$= \int_0^6 \left(2x - (x^2 - 4x)\right)dx = \int_0^6 (-x^2 + 6x)\,dx$$

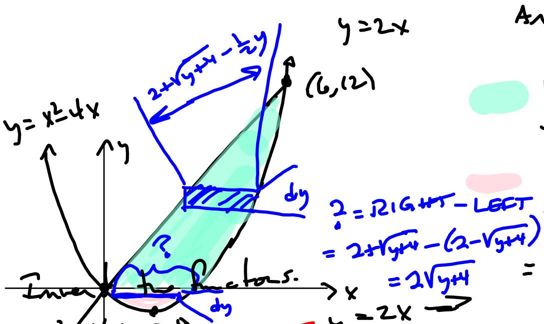
$$= \left[-\frac{x^3}{3} + \frac{6x^2}{2}\right]_0^6 = \left[-\frac{x^3}{3} + 3x^2\right]_0^6$$

$$= -\frac{216}{3} + 3(36) - (0-0) = -72 -$$

$$= -72 + 108 = \boxed{36} = \text{Area.}$$

Can we do it sideways?
Yes. Yes, we can.

$$\text{Area} = \int_a^b (\text{Right} - \text{Left})\,dy$$



$? = \text{RIGHT} - \text{LEFT}$

$= 2 + \sqrt{y+4} - (2 - \sqrt{y+4})$

$= 2\sqrt{y+4}$

$y = x^2 - 4x$ $(2,-4)$ $\quad x = 2 + \sqrt{y+4}$ $\quad y = 2x \longrightarrow$

$x^2 - 4x + 2^2 = y + 4$ $\quad$ Vertex: $y = (x-2)^2 - 4$ $\quad x = \frac{1}{2}y$

$(x-2)^2 = y + 4$ $\Rightarrow y = (x-2)^2 - 4$

$(h,k) = (2,-4)$

$x - 2 = \pm\sqrt{y+4}$

$x = 2 \pm \sqrt{y+4}$

$$\int_0^{12} \left(2 + \sqrt{y+4} - \frac{1}{2}y\right)dy \; +$$

$$\int_{-4}^{0} \left(2 + \sqrt{y+4} - (2 - \sqrt{y+4})\right)dy$$

$$= \int_0^{12} \left(2 + (y+4)^{\frac{1}{2}} - \frac{1}{2}y\right)dy$$

$$+ \int_{-4}^{0} 2\left(\frac{2}{3}\right)(y+4)^{\frac{3}{2}}\,dy \Big]_{-4}$$

$$= 2\left[(12) - \frac{2}{3}(16)^{\frac{3}{2}} - \frac{1}{4}\left(\frac{1}{2}y^2\right)\right]_0^{12}$$

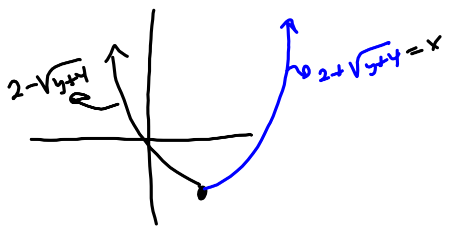
$$+ \frac{4}{3}(4)^{3/2}$$

$$= 24 + \frac{2}{3}(4)^3 - \frac{1}{4}(144) + \frac{4}{3}(2)^3$$

$$= 24 + \frac{2}{3}(64) + 36 + \frac{4}{3}(8)$$

$$= 24 + \frac{128}{3} + 36 + \frac{32}{3}$$

$$= 60 + \frac{160}{3}$$

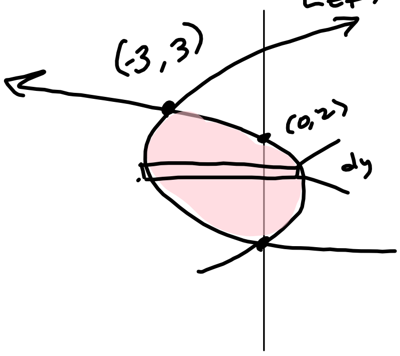


Not sure where I went wrong, but Maple says I wrote the right integral.

Area bdd by $x = y^2 - 4y$ & $x = 2y - y^2$

$x = y(y-4)$ & $x = -y(y-2)$

LEFT RIGHT



So $\int_0^{?=3} \left(-y^2+2y - (y^2-4y)\right) dy$

?: $y^2-4y = -y^2+2y$

$2y^2 - 6y = 0$

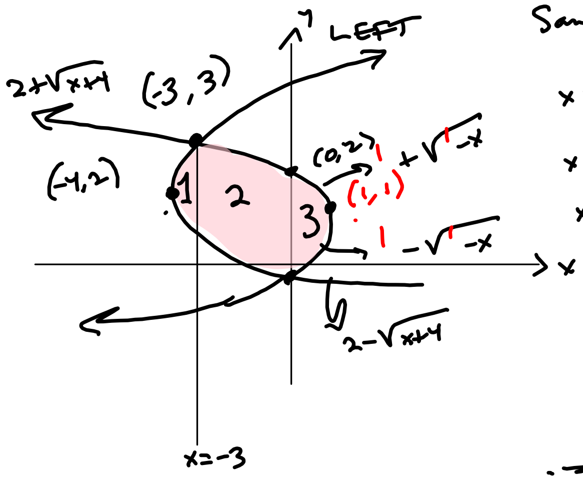
$2y(y-3) = 0$

$y = 3$ OR $y = 0$

$x = y^2 - 4y = 3^2 - 4(3) = 9 - 12 = -3$

$= \int_0^3 (-2y^2+6y)\,dy = \left[-\frac{2y^3}{3} + \frac{6y^2}{2}\right]_0^3 = \left[-\frac{2}{3}y^3 + 3y^2\right]_0^3$

$= -\frac{2}{3}(3^3) + 3(3)^2 = -2(9) + 27 = -18 + 27 = \boxed{9 = \text{Area}}$



Same thing but wrt x.

$x=y^2-4y$

$x+4=y^2-4y+2^2$

$x+4=(y-2)^2=x+4$

$y-2=\pm\sqrt{x+4}$

$y=2\pm\sqrt{x+4}$

$3^2-4(3)$
$=9-12=-3$

$x=-y^2+2y=-(y^2-2y+1^2)+1$

$x-1=-(y-1)^2$

$(y-1)^2=1-x$

$y-1=\pm\sqrt{1-x}$

$y=1\pm\sqrt{1-x}$

$$\int_{-4}^{-3}\left(2+\sqrt{x+4}-(2-\sqrt{x+4})\right)dx$$

$$+\int_{-3}^{0}\left(1+\sqrt{4-x}-(2-\sqrt{x+4})\right)dx$$

$$+\int_{0}^{4}\left(2+\sqrt{4-x}-(2-\sqrt{4-x})\right)dx$$

$$=\int_{-4}^{-3}2\sqrt{x+4}\,dx+\int_{-3}^{0}\left(\sqrt{4-x}+\sqrt{x+4}-1\right)dx+\int_{0}^{4}2\sqrt{4-x}\,dx$$

$y = x^2 - 4x$ invert this

Solve $x^2 - 4x = y$ for x

$x^2 - 4x - y = 0$

$a = 1,\ b = -4,\ c = y$

$b^2 - 4ac = 4^2 - 4(1)(y) = 16 - 4y$

$x = \dfrac{4 \pm \sqrt{16-4y}}{2} = \dfrac{4 \pm 2\sqrt{4-y}}{2} = 2 \pm \sqrt{4-y} = x$

$-x^2 + 2x = y$

$-x^2 + 2x - y = 0$

$a = -1,\ b = 2\ c = -y$

$b^2 - 4ac = 4 - 4(-1)(-y)$

$= 4 - 4y = 4(1-y)$

$-x^2 + 2x = y$

$-(x^2 + 2x + 1^2) = y - 1$

$(x+1)^2 = 1 - y$

$x + 1 = \pm\sqrt{1-y}$

$x = -1 \pm \sqrt{1-y}$