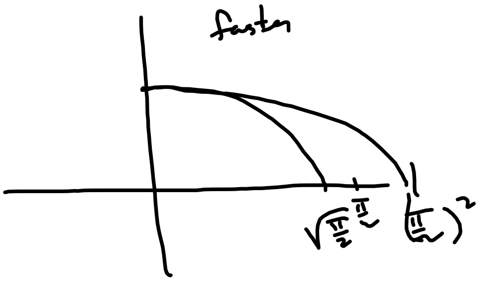
$$\int_0^{.5} \underbrace{\cos(x^2)}_{\text{Decreases faster}}\,dx \qquad \int_0^{.5} \cos(\sqrt{x})\,dx$$



$$\sqrt{x} = \frac{\pi}{2}$$
$$x = \left(\frac{\pi}{2}\right)^2$$

$$f(x) = \begin{cases} 2 & \text{if } x < 2 \\ x & \text{if } x \geq 2 \end{cases}$$

$$\int_0^4 f(x) = \int_0^2 2\,dx + \int_2^4 x\,dx = \Big[2x\Big]_0^2 + \left[\frac{x^2}{2}\right]_2^4$$

$$= 4 + \frac{4^2}{2} - \frac{2^2}{2} = 4 + 6 = 10$$

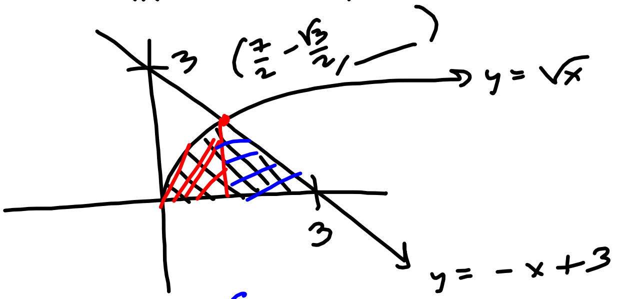
$\int \cos(t)\sqrt{\sin(t)+77}\,dt$

$u = \sin(t) + 77 \Rightarrow$

$du = \cos(t)\,dt$

$\Rightarrow \int \sqrt{u}\,du = \dfrac{u^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{2}{3}(\sin(t)+77)^{\frac{3}{2}}$

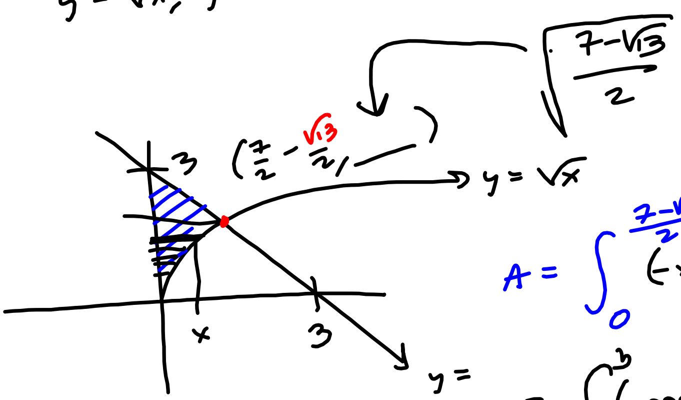
Find the area bounded by $y=\sqrt[3]{x}$, $y=-x+3$, $y=0$



$$-x+3=\sqrt{x}$$
$$x^2-6x+9=x$$
$$x^2-7x+9=0$$
$$x^2-7x+\left(\frac{7}{2}\right)^2+9-\frac{49}{4}$$
$$=\left(x-\frac{7}{2}\right)^2+\frac{36-49}{4}=0$$
$$\left(x-\frac{7}{2}\right)^2=\frac{13}{4}$$
$$x=\frac{7}{2}\pm\frac{\sqrt{13}}{2}$$
$$y=\sqrt{\frac{7-\sqrt{13}}{2}}$$

$$\text{Area}=\int_0^{\frac{7-\sqrt{13}}{2}}\sqrt{x}\,dx+\int_{\frac{7-\sqrt{13}}{2}}^{3}(-x+3)\,dx$$

$y=\sqrt[3]{x}$, $y=-x+3$, $x=0$



$$A=\int_0^{\frac{7-\sqrt{13}}{2}}(-x+3)\,dx-\int_0^{\frac{7-\sqrt{13}}{2}}\sqrt{x}\,dx$$

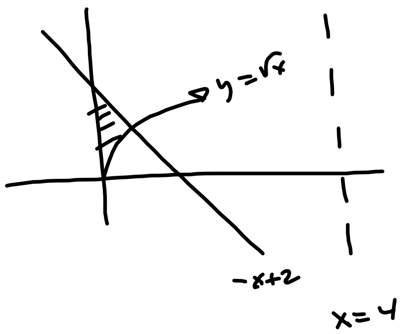
$$=\int_a^b(\text{upper}-\text{lower})\,dx$$

Can you do it with a dy?

$$\int_0^{\sqrt{\frac{7-\sqrt{13}}{2}}}y^2\,dy+\int_{\sqrt{\frac{7-\sqrt{13}}{2}}}^{3}(-y+3)\,dy$$

Scratch: $y=\sqrt{x}$
$x=y^2$
$y=-x+3$
$-x=y-3$
$x=-y+3$

Rotate the region about the line x = 4



$y = -x + b$

$y = \sqrt{x}$

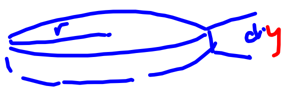
$\sqrt{x} = -x + b$

$x = x^2 - 2bx + b^2$

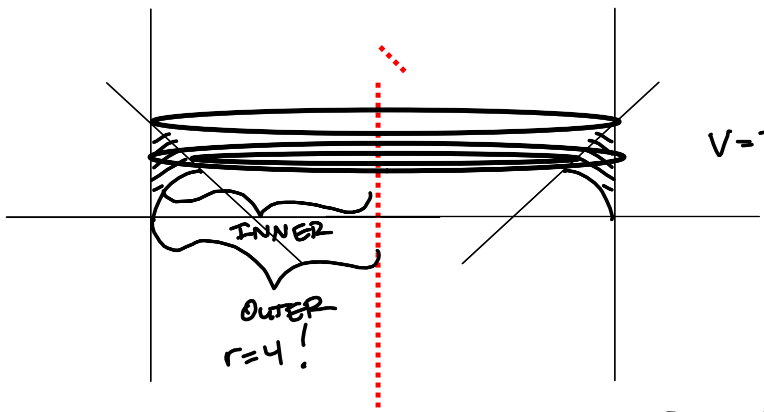
$\Rightarrow x^2 - (2b+1)x + b^2$

$y = -x + 2$

Volume of disc of radius r & thickness dx is



$V = \pi r^2 h = \pi r^2 dy$!



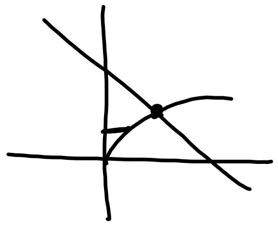
$V = \pi \int \left(\text{OUTER}^2 - \text{INNER}^2\right) dy$

$y = \sqrt{x} \rightarrow x = y^2$

$y = -x+2 \rightarrow x = -y+2$

→ OUTER = 4 !

$\int_0$



$\sqrt{x} = -x+2$

$x = x^2-4x+4$

$x^2-5x+4 = 0$

$(x-4)(x-1) = 0$

x=4 extraneous

$\sqrt{4} = -4+2$

$2 = -2$ NO

$\sqrt{1} = -1+2$?

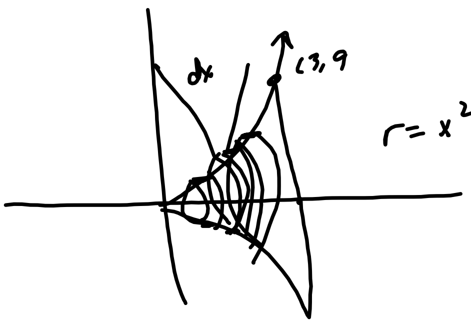
$1 = 1$ ✓

$x=1 \rightarrow y = -x+2 = 1$

$\pi \int_0^1 \left(4^2 - (4-y^2)^2\right) dy + \pi \int_1^2 \left(4^2 - (4-(-x+2))^2\right) dy$

$y = x^2$, $x=0$, $x=3$ about x-axis?

$y = -1$?



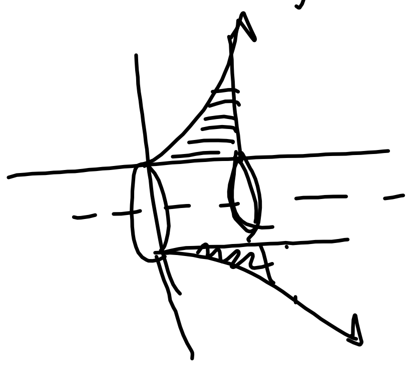
$r = x^2$

$$\pi\int_0^3 r^2\,dx = \pi\int_0^3 (x^2)^2\,dx$$

$$= \pi\int_0^3 x^4\,dx = \pi\frac{3^5}{5}$$

= volume.

About $y = -1$



$r = x^2 - (-1) = x^2 + 1 =$ OUTER

Inner $= 1$, so

$$V = \pi\int_0^3 (\text{outer}^2 - \text{inner}^2)\,dx$$

$$= \pi\int_0^3 \left((x^2+1)^2 - 1^2\right)dx$$

$$= \pi\int_0^3 (x^4 + 2x^2 + 1 - 1)\,dx$$

$$= \pi\left[\frac{x^5}{5} + \frac{2}{3}x^3\right] = \pi\left[\frac{3^5}{5} + \frac{2}{3}\cdot 3^3\right]$$

etc.