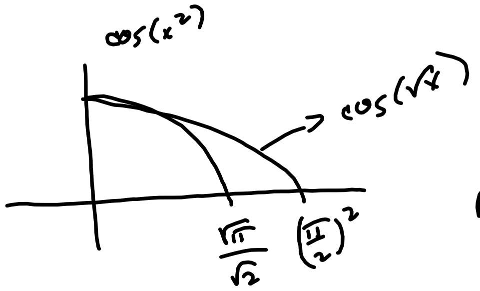
$\int_0^{.5} \cos(x^2)\,dx$ -vs- $\int_0^{0.5} \cos(\sqrt{x})\,dx$

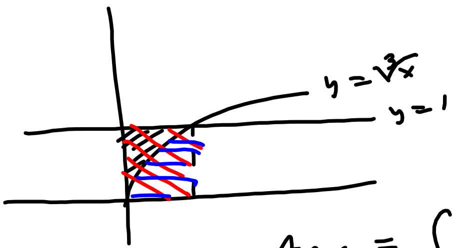


$\sqrt{x} = \frac{\pi}{2}$

$x = \frac{\pi^2}{2^2}$

Return to th. 3

$$\int_a^b \frac{6 + \cos^2(x)}{\cos^2(x)}\,dx = \int_a^b \left(6\sec^2(x) + 1\right)dx = \Big[\tan(x) + x\Big]_a^b$$



$$\text{Area} = \int_0^1 (1 - \sqrt[3]{x})\,dx$$
$$= \int (\text{upper} - \text{lower})$$

$$\int \sec^2(x)\tan^2(x)\,dx = ?$$
$$= \int (\sec(x)\tan(x))^2\,dx$$

$\int u^n\,du$

$u = \sec(x) \rightarrow$

$du = \sec(x)\tan(x)$

$$\rightarrow \int \underbrace{\sec(x)}_{u}\,\underbrace{(\sec(x)\tan(x))\,\tan(x)\,dx}_{du}$$

oops!

$u = \tan(x) \rightarrow$

$du = \sec^2(x)\,dx \rightarrow$

$$\int \tan^2(x)\left[\sec^2(x)\,dx\right] = \int u^2\,du = \frac{u^3}{3} + C$$
$$= \frac{\tan^3(x)}{3} + C$$

M1 & M2 from class, yesterday:

$\boxed{M1}$

$$\int_0^1 x\sqrt{x^2+5}\,dx = \frac{1}{2}\int_0^1 \sqrt{x^2+5}\,(2x\,dx)$$

$$= \frac{1}{2}\left[\frac{2}{3}(x^2+5)^{\frac{3}{2}}\right]_0^1 = etc.$$

$u = x^2+5$

$du = 2x\,dx$

$\underline{dx} = \frac{du}{2x}$

$$\int u^{\frac{1}{2}}du = \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C$$

$\boxed{M2}$

sidestepping

$x=0 \rightarrow u=5$

$x=1 \Rightarrow u=6$

$$\frac{1}{2}\int_5^6 u^{\frac{1}{2}}du = \frac{1}{2}\left[\frac{2}{3}u^{\frac{3}{2}}\right]_5^6$$

$$\int_0^1 x\sqrt{x^2+5}\,dx = \int_{5=u}^{6=u} \cancel{x}\sqrt{u}\,\frac{du}{2\cancel{x}} = \frac{1}{2}\int_5^6 u^{\frac{1}{2}}du$$

$$\int_0^4 f(x)\,dx = 4 \longrightarrow$$

$$\int_0^2 x f(x^2)\,dx = ? \quad = \int_{0=u}^{4=u} x f(u) \frac{du}{2x} = \frac{1}{2}\int_0^4 f(u)\,du$$

$$= \frac{1}{2}\left[4\right] = 2$$

$u = x^2$ $\qquad x=0 \Rightarrow u=0$

$du = 2x\,dx$ $\qquad x=2 \Rightarrow u=4$

$\frac{du}{2x} = dx$

$$g(x) = \int_{\sin(x)}^{x^3} \frac{1+t}{\sqrt{t^2-3}}\,dt = \int_a^0 f(t)\,dt$$

$$= \int_{\sin(x)}^{0} f(t)\,dt + \int_0^{x^3} f(t)\,dt$$

$$= -\int_0^{\sin(x)} f(t)\,dt + \int_0^{x^3} f(t)\,dt \longrightarrow$$

$$g'(x) = f(\sin(x))\,\frac{d}{dx}\sin(x) + f(x^3)\,\frac{d}{dx}x^3$$

Let $h(x) = \int_0^x f(t)\,dt \longrightarrow$

$$h'(x) = f(x) \longrightarrow$$

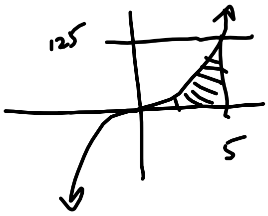
$$h(x^3) = \int_0^{x^3} f(t)\,dt = f(x^3)\cdot 3x^2$$

$$h(x) = \sin(x) \longrightarrow h'(x) = \cos(x)$$

$$h(x^3) = \sin(x^3) \longrightarrow h'(x^3) = \cos(x^3)\cdot 3x^2$$

Estimate

$$\int_0^5 x^3\,dx$$



$m = 0 = \min_{x\in[0,5]} \{x^3\}$

$M = 125 = \max_{x\in[0,5]} \{x^3\}$

$b-a = 5$

$0(5) \le \int_0^5 x^3\,dx \le 125 \cdot 5 = 625$

$f(x) = x - 1$

$[a,b] = [-6,4]$

$n = 5$ rectangles

$\frac{b-a}{n} = \Delta x = \frac{4-(-6)}{5} = \frac{10}{5} = 2$

Area $\approx \Delta x \sum_{k=1}^{5} f(x_k)$

$x_k = a + k \cdot \Delta x$ right $= -6 + 2k$

$x_k = a + (k-1)\Delta x$ left

$2 \sum_{k=1}^{5} f(2k-6) = 2 \sum_{k=1}^{5} ((2k-6) - 1)$

$2 \sum_{k=1}^{5} (x_k - 1)$

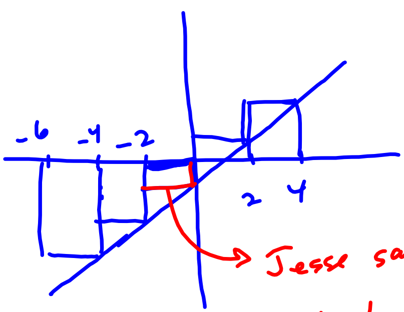
$= 2 \sum_{k=1}^{5} (2k-7) = 2 \sum_{k=1}^{5} 2k - 2 \sum_{k=1}^{5} 7$

$= 4 \sum_{k=1}^{5} k - 14 \sum_{k=1}^{5} 1$

$= 4\left(\frac{5(5+1)}{2}\right) - 14[5]$

$= 4(15) - 70$

$= 60 - 70 = -10$



Jesse says I missed this, but I was just testing you.

$$\int_{-6}^{2} f(x)\,dx + \int_{2}^{3} f(x)\,dx - \int_{-6}^{-3} f(x)\,dx$$

$$= \int_{-6}^{3} f - \int_{-6}^{-3} f = \int_{-3}^{3} f$$

$$\int x^2\sqrt{x^3+9}\,dx = \frac{1}{3}\int (x^3+9)^{\frac{1}{2}}(3x^2)\,dx$$

$$= \frac{1}{3}\left[\frac{2}{3}(x^3+9)^{\frac{3}{2}}\right] + C$$