$$\int \cos^2(x)\,dx = \int \frac{1+\cos(2x)}{2}\,dx = \frac{1}{2}\int dx + \frac{1}{2}\int \cos(2x)\,dx$$

$$\left(\frac{d}{dx}\left[\sin(2x)\right] = \cos(2x)\cdot 2 = 2\cos(2x) \implies \int 2\cos(2x)\,dx = \sin(2x) + C \right)$$

$$= \frac{1}{2}x + \frac{1}{2}\cdot\frac{1}{2}\int \cos(2x)\cdot 2\,dx = \frac{1}{2}x + \frac{1}{4}\sin(2x) + C$$

$$\frac{d}{dx}\left[\frac{1}{2}x + \frac{1}{4}\sin(2x) + C\right] = \frac{1}{2} + \frac{1}{4}\left(\cos(2x)\cdot 2\right)$$

$$= \frac{1+\cos(2x)}{2} = \cos^2(x)$$

Requires power-reduction formula from trigonometry.

NET CHANGE §4.4

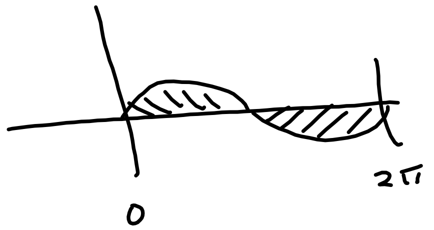
$$\int_a^b F'(x)\,dx = F(b) - F(a)$$

The definite integral of a rate is the net change.

TOTAL CHANGE:

$$\int_a^b |f'(x)|\,dx$$

$\sin(x)$



$$\int_0^{2\pi} \sin(x)\,dx = -\cos(x)\Big]_0^{2\pi}$$
$$= -\cos(2\pi) - (-\cos(0))$$
$$= -1 - (-1) = 0$$

$$\int_0^{2\pi} |\sin(x)|\,dx =$$

$$|\sin(x)| = \begin{cases} \sin(x) & \text{if } \sin(x) \geq 0 \\ -\sin(x) & \text{if } \sin(x) < 0 \end{cases} = \begin{cases} \sin(x) & \text{if } 0 \leq x \leq \pi \\ -\sin(x) & \text{if } \pi < x < 2\pi \end{cases}$$

$$= \int_0^{\pi} \sin(x)\,dx + \int_{\pi}^{2\pi} -\sin(x)\,dx = -\cos(x)\Big]_0^{\pi} - (-\cos(x))\Big]_{\pi}^{2\pi}$$

$$= -\cos(\pi) - (-\cos(0)) + \cos(2\pi) - \cos(\pi)$$

$$= -(-1) + 1 + 1 - (-1) = 4$$

$$\int_1^4 \frac{4+6u}{\sqrt{u}}\,du = \int_1^4 \left(\frac{4}{\sqrt{u}} + \frac{6u}{\sqrt{u}}\right)du = \int_1^4 \left(4u^{-\frac{1}{2}} + 6u^{\frac{1}{2}}\right)du$$

$$= \left[4\frac{u^{\frac{1}{2}}}{\frac{1}{2}} + 6\frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^4 = \left[8u^{\frac{1}{2}} + 4u^{\frac{3}{2}}\right]_1^4 = \left(8(4)^{\frac{1}{2}} + 4(4)^{\frac{3}{2}}\right)$$

$$-\left(8(1)^{\frac{1}{2}} + 4(1)^{\frac{3}{2}}\right) = 16 + 4(8) - (8+4) = 16+32-12 = \boxed{36}$$

$$4^{\frac{3}{2}} = \left(4^{\frac{1}{2}}\right)^3 = 2^3 = 8$$

$$= \left(4^3\right)^{\frac{1}{2}} = (64)^{\frac{1}{2}} = 8$$

S 4.5 - u-Substitution

$$\int \frac{\cos(x)}{\sin^2(x)}\,dx = \int \frac{\cos(x)}{\sin(x)} \cdot \frac{1}{\sin(x)}\,dx = \int \cot(x)\csc(x)\,dx$$

$$= \int \csc(x)\cot(x)\,dx = -\csc(x) + C$$

CHECK: $\frac{d}{dx}\left[-\csc(x)\right] = -\frac{d}{dx}\left[\csc(x)\right] = -\left(-\csc(x)\cot(x)\right)$ ✓

$$\frac{d}{dx}\left[\sin(x)\right] = \cos(x)$$

$$\Rightarrow d(\sin(x)) = \cos(x)\,dx$$

$$\Rightarrow \int \frac{\cos(x)}{\sin^2(x)}\,dx = \int \frac{\cos(x)\,dx}{\sin^2(x)} = \int \frac{d(\sin(x))}{\sin^2(x)}$$

$$= \int \sin^{-2}(x)\, d(\sin(x))$$

$$= \int u^{-2}\,du, \text{ where } u = \sin(x) \text{ \& } du = \cos(x)\,dx$$

$$= \frac{u^{-1}}{-1} + C = -\frac{1}{\sin(x)} + C = -\csc(x) + C$$

if you can "see" the cos(x) as the derivative of the sin(x) & you know the chain Rule.

$$\int (x^2+7x)^{\frac{1}{3}}(2x+7)\,dx = \int u^{\frac{1}{3}}\,du = \frac{3}{4}u^{\frac{4}{3}} + C$$

Let $u = x^2+7x \Rightarrow du = (2x+7)\,dx$

$$= \boxed{\frac{3}{4}(x^2+7x)^{\frac{4}{3}} + C}$$

$$\int \frac{x^3}{(x^4-5)^2}\,dx = \frac{1}{4}\int (x^4-5)^{-2}(4x^3\,dx)$$

Let $u = x^4-5$

$\Rightarrow du = 4x^3\,dx \quad \longrightarrow$

$\hookrightarrow \frac{du}{4x^3} = dx$

$$\int \frac{x^3}{u^2}\left(\frac{du}{4x^3}\right) = \frac{1}{4}\int \frac{du}{u^2} = \frac{1}{4}\int u^{-2}\,du = \frac{1}{4}\,\frac{u^{-1}}{-1} + C$$

$$= -\frac{1}{4u} + C = -\frac{1}{4}\cdot\frac{1}{(x^4-5)} + C = \boxed{\frac{-1}{x^4-5} + C}$$

$$\int x\sqrt{1-x^2}\,dx \quad = \frac{1}{-2}\int (1-x^2)^{\frac{1}{2}}(-2x\,dx)$$

$u = 1-x^2 \Rightarrow$

$du = -2x\,dx \Rightarrow$

$dx = \frac{du}{-2x}$

$$\Rightarrow \int x\,(1-x^2)^{\frac{1}{2}}\left(\frac{du}{-2x}\right) = \int u^{\frac{1}{2}}\frac{du}{-2} = -\frac{1}{2}\int u^{\frac{1}{2}}du, \text{ etc.}$$

$$\int_1^2 x\sqrt{x-1}\,dx = \int_1^2 x\,u^{\frac{1}{2}}\,dx = \int_{1=x}^{2=x} x\,u^{\frac{1}{2}}\,du$$

$u = x-1 \Rightarrow u+1 = x \Rightarrow \int_{1=x}^{2=x} (u+1)u^{\frac{1}{2}}\,du$

$du = dx$

2 ways to handle limits of integration:

1.
$$= \int_{x=1}^{x=2} \left(u^{\frac{3}{2}} + u^{\frac{1}{2}}\right) du = \left[\frac{2}{5}u^{\frac{5}{2}} + \frac{2}{3}u^{\frac{3}{2}}\right]_{x=1}^{x=2}$$

$$= \left[\frac{2}{5}(x-1)^{\frac{5}{2}} + \frac{2}{3}(x-1)^{\frac{3}{2}}\right]_{x=1}^{x=2} = \frac{2}{5}(1)^{\frac{5}{2}} + \frac{2}{3}(1)^{\frac{3}{2}} - \left(\frac{2}{5}(0) + \frac{2}{3}(0)\right)$$

$$= \frac{2}{5}\cdot\frac{3}{3} + \frac{2}{3}\cdot\frac{5}{5} = \boxed{\frac{16}{15}}$$

2.
$$\int_1^2 x\sqrt{x-1}\,dx$$

$u = x-1 \Rightarrow x = u+1$ $\qquad x=1 \Rightarrow u = 0$

$du = dx$ $\qquad x=2 \Rightarrow u = 1$

$$= \int_0^1 (u+1)u^{\frac{1}{2}}\,du = \left[\frac{2}{5}u^{\frac{5}{2}} + \frac{2}{3}u^{\frac{3}{2}}\right]_0^1$$

$$= \frac{2}{5}(1)^{\frac{5}{2}} + \frac{2}{3}(1)^{\frac{3}{2}} - (0+0) = \boxed{\frac{16}{15}}$$

Evaluate $\int_{-2}^{2}(x+6)\sqrt{4-x^2}\,dx$ by writing it as a sum of two integrals and interpreting one of those integrals in terms of an area.

$$I = \int_{-2}^{2}\left(x\sqrt{4-x^2} + 6\sqrt{4-x^2}\right)dx$$

$$= \underbrace{\int_{-2}^{2} x\sqrt{4-x^2}\,dx}_{0} + 6\underline{\int_{-2}^{2}\sqrt{4-x^2}\,dx}$$

$$= 0$$

$y=\sqrt{4-x^2}$ = Top $\frac{1}{2}$ of circle of radius $r=2$

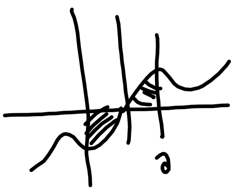
-2 2

$$\text{Area} = \frac{1}{2}\left[\pi\cdot 2^2\right]$$

$$= 2\pi$$

$x\sqrt{4-x^2}$ is an ODD function.

$$-x\sqrt{4-(-x)^2} = -x\sqrt{4-x^2}$$



-2 2

$$\int_{-2}^{2}\text{ODD} = 0$$

$$\Rightarrow I = 6(2\pi) = 3\pi \rightarrow 12\pi$$

b/c Jesse can do arithmetic!