$\int_a^b = -\int_b^a$

$y=\sqrt{9-x^2}$ = Top $\frac{1}{2}$ of circle of radius $r=3$.

$y^2 = 9-x^2$

$x^2+y^2=3^2$

$\int_a^b x\,dx = \frac{b^2-a^2}{2}$ & $\int_0^{\frac{\pi}{2}} \cos(x)\,dx = 1$

Find $\int_0^{\frac{\pi}{2}} (4\cos(x)-3x)\,dx = 4\int_0^{\frac{\pi}{2}} \cos(x)\,dx - 3\int_0^{\frac{\pi}{2}} x\,dx$

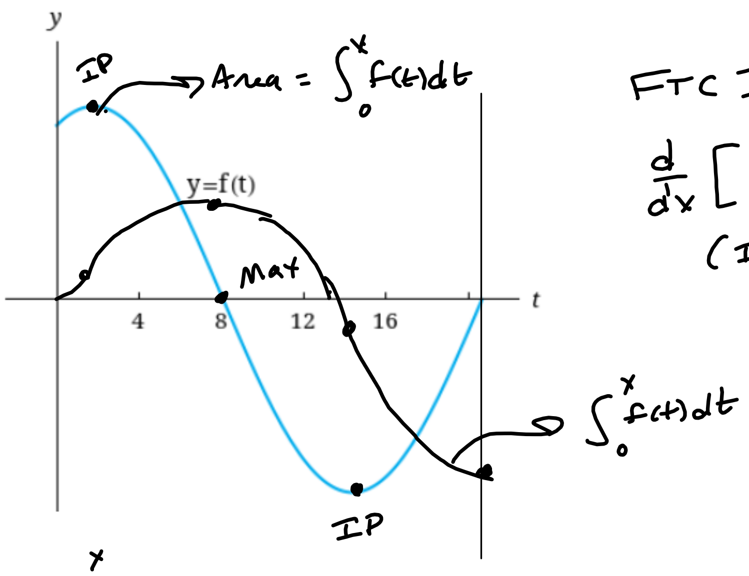
$= 4-3\left[\frac{(\frac{\pi}{2})^2-0^2}{2}\right] = 4-3\left(\frac{\pi^2}{8}\right) = 4-\frac{3\pi^2}{8}$

$\int_a^b x\,dx = \left[\frac{x^2}{2}\right]_a^b = \frac{b^2}{2}-\frac{a^2}{2}$

FTC II

$f(x)=\int_a^x g(t)\,dt = \int_a^x f'(x)\,dt$

$\int^x t^2\,dt$



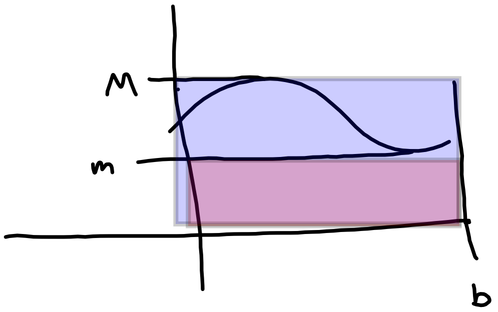
FTC I:

$\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(t)$

(If f is cnts on $[a,b]$, & $x \in (a,b)$)

The derivative of the antiderivative is the integrand.

$\min_{x \in [a,b]} f(x) = m$ & $\max_{x \in [a,b]} = M \longrightarrow$



Area is $M(b-a)$ (blue)

" " $m(b-a)$ (pink)

This means

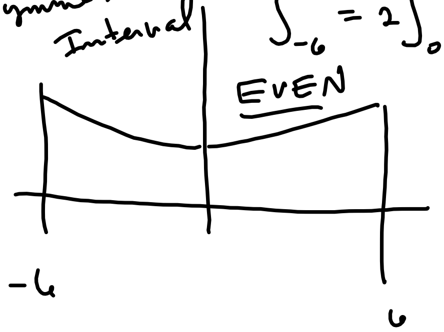
$$m(b-a) \le \int_a^b f(x)\,dx \le M(b-a)$$

Find Bounds on $\sqrt{4+x^2}$ on $[-6,6]$

Symmetric Interval

$$\int_{-6}^{6} = 2\int_{0}^{6}$$

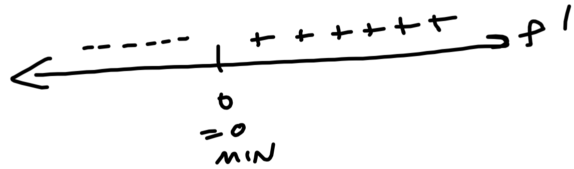
EVEN



$(4+x^2)^{\frac{1}{2}} = f(x) \Rightarrow$

$f'(x) = \frac{1}{2}(x^2+4)^{-\frac{1}{2}}(2x)$

$= \dfrac{x}{\sqrt{x^2+4}}$ SET 0 $\rightarrow x=0$



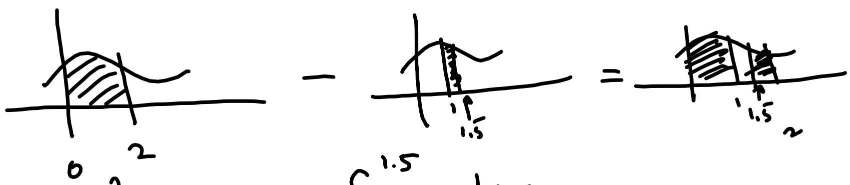
f is even:

$f(-x) = \sqrt{(-x)^2+4} = \sqrt{x^2+4} = f(x)$

$m = f(0) = 2$

$M = f(6) = \sqrt{4+36} = \sqrt{40} = 2\sqrt{10}$

$\therefore \quad 2(12) \le \int_{-6}^{6} \sqrt{x^2+4}\,dx \le 2\sqrt{10}\,(12) = 24\sqrt{10}$



$\int_{0}^{2} f(x)dx \quad - \int_{1}^{1.5} f(x)\,dx =$

FTC I and II Proofs on Monday.

$$\frac{d}{dx}\left[\int_1^x \sin(t^2+2)\,dt\right] = \sin(x^2+2) = f'(x)$$

Let $f(x) = \int_1^x \sin(t^2+2)\,dt$.

Then $f(\cos(x)) \Longrightarrow \frac{df}{dx} = \frac{df}{d\cos(x)} \cdot \frac{d\cos(x)}{dx}$

$f(u(x)) \Rightarrow \frac{d}{dx}\left[f(u(x))\right] = \frac{df}{du} \cdot \frac{du}{dx}$

$u(x) = \cos(x)$

$$f(\cos(x)) = \int_1^{\cos(x)} \sin(t^2+2)\,dt$$

$\Longrightarrow \frac{d}{dx}\left[f(\cos(x))\right] = \frac{df}{du} \cdot \frac{du}{dx} = \sin(\cos^2(x)+2)(-\sin(x))$

Chain Rule version of FTC I.

$$\frac{d}{dx}\left[\int_0^{\tan(x)} \frac{t^2-7}{\sin(t)+2}\,dt\right] = \left(\frac{\tan^2(x)-7}{\sin(\tan(x))+2}\right)\sec^2(x) \quad !$$

$$\frac{d}{dx}\left[\int_{x^2}^{\cos(x)} \frac{t^2+1}{t^2+7}\,dt\right] = \frac{d}{dx}\left[\int_{x^2}^{0} \frac{t^2+1}{t^2+7}\,dt + \int_0^{\cos(x)} \frac{t^2+1}{t^2+7}\,dt\right]$$

$$= \frac{d}{dx}\left[-\int_0^{x^2} \frac{t^2+1}{t^2+7}\,dt + \int_0^{\cos(x)} \frac{t^2+1}{t^2+7}\,dt\right]$$

$$= \left(-\frac{(x^2)^2+1}{(x^2)^2+7}\right)(2x) + \left(\frac{\cos^2(x)+1}{\cos^2(x)+7}\right)(-\sin(x))$$

MEAN VALUE THEOREM For Monday.

f cnt^{s} on $[a,b]$ & f difb^{l} on (a,b) $\longrightarrow$

$\exists\, c \in (a,b) \ni f'(c) = \dfrac{f(b)-f(a)}{b-a}$

V2: $\exists\, c \in (a,b) \ni \underbrace{f(b)-f(a)}_{\text{net change in } f} = f'(c)(b-a)$