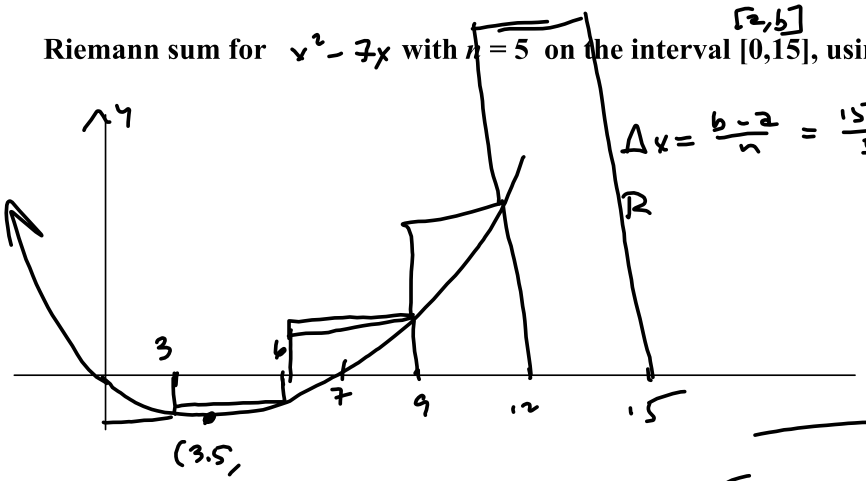
$$\sum_{k=1}^{n} k = 1 + 2 + 3 + \cdots + n$$

$$\sum_{k=1}^{n} k^2 = 1^2 + 2^2 + 3^2 + \cdots + n^2$$

$$\sum_{k=1}^{n} \sin\left(\frac{k\pi}{6}\right) = \underset{k=1}{\sin\left(\frac{\pi}{6}\right)} + \underset{k=2}{\sin\left(\frac{2\pi}{6}\right)} + \underset{k=3}{\sin\left(\frac{3\pi}{6}\right)} + \cdots + \sin\left(\frac{n\pi}{6}\right)$$

Riemann sum for $x^2 - 7x$ **with** $n = 5$ **on the interval [0,15], using right endpoints.**

[a,b]



$\Delta x = \frac{b-a}{n} = \frac{15-0}{5} = 3 = \Delta x$

$\text{Area} \approx \sum_{k=1}^{5} f(x_k)\Delta x = \Delta x \sum_{k=1}^{5} f(x_k)$

$a=0, b=15$

$f(x_1)\Delta x + \dots + f(x_5)\Delta x = \Delta x\,(f(x_1) + \dots + f(x_5))$

$x_1 = a + \Delta x = 0 + 3$

$x_3 = a + 3\Delta x = 3(3) = 9$

$a + k\Delta x = x_k$

$0 + k\cdot 3 = 3k$

$\text{Area} \approx 3\sum_{k=1}^{5} f(x_k) = 3\sum_{k=1}^{5}(x_k^2 - 7x_k) = 3\sum_{k=1}^{5}\left((3k)^2 - 7(3k)\right)$

$= 3\sum_{k=1}^{5}(9k^2 - 21k) = 3\left[\sum_{k=1}^{5}(9k^2) - \sum_{k=1}^{5} 21k\right]$

$= 3\sum_{k=1}^{5} 9k^2 - 3\sum_{k=1}^{5} 21k = 27\sum_{k=1}^{5} k^2 - 63\sum_{k=1}^{5} k$

$= 27\left[1 + 2^2 + 3^2 + 4^2 + 5^2\right] - 63\left[1+2+3+4+5\right]$

$= 27\left[\frac{5(5+1)(2(5)+1)}{6}\right] - 63\left[15\right]$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\boxed{= \frac{n^3 + \sim}{3}}$$

For $\int_0^{15} (x^2 - 7x)\,dx$

$$\sum_{k=1}^{5} k = \frac{5(6)}{2} = 15$$

$$\sum_{k=1}^{k} k = \frac{n(n+1)}{2}$$

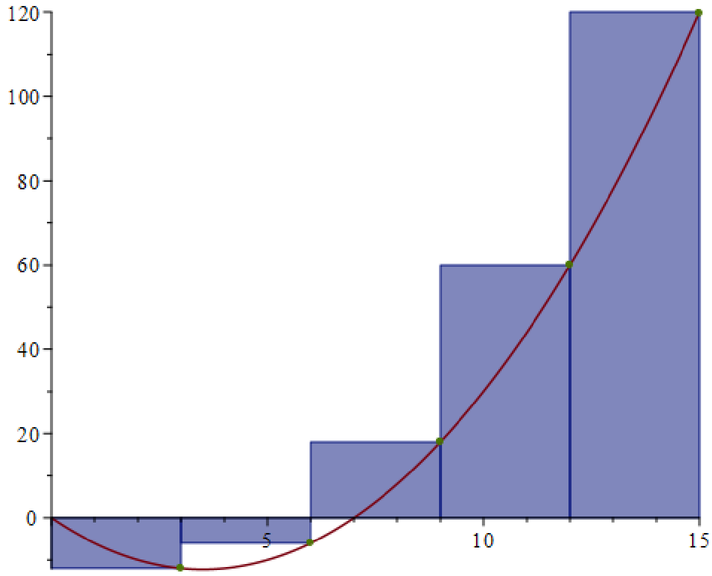
Scratch

$$\to 27\left[\frac{30(11)}{6}\right] - 63\left[15\right] = 27(55) - 945 = \boxed{540}$$

Doesn't Check

Yes, it does, knucklehead.

630
3.5



A right Riemann sum approximation of $\int_0^{15} f(x)\,dx$, where $f(x) = x^2 - 7x$ and the partition is uniform. The approximate value of the integral is 540.0000000. Number of subintervals used: 5.

Let's find the ACTUAL (signed) area under $x^2 - 7x$

1. By Limit Defin
2. By cheating, with FTC II.

$f(x) = x^2 - 7x$, n = arbitrary, $[a,b] = [0,15]$

$\Delta x = \frac{b-a}{n} = \frac{15}{n}$

$x_k = a + k\Delta x = 0 + \frac{15}{n}k$,

$\text{Area} = \lim_{n\to\infty} \Delta x \sum_{k=1}^{n} f(x_k)$:

$$\Delta x \sum_{k=1}^{n} f(x_k) = \frac{15}{n} \sum_{k=1}^{n} (x_k^2 - 7x_k) = \frac{15}{n} \sum_{k=1}^{n} \left[\left(\frac{15}{n}k\right)^2 - 7\left(\frac{15}{n}k\right)\right]$$

$$= \frac{15}{n}\left[\sum_{k=1}^{n} \frac{15^2}{n^2}k^2 - \sum_{k=1}^{n} 7\left(\frac{15}{n}k\right)\right]$$

$$= \frac{15}{n}\left[\frac{15^2}{n^2}\sum_{k=1}^{n} k^2 - \frac{7(15)}{n}\sum_{k=1}^{n} k\right]$$

$$= \frac{15^3}{n^3}\left(\frac{n^3 + \cdots}{3}\right) - \frac{15}{n}\cdot\frac{7(15)}{n}\left(\frac{n^2 + \cdots}{2}\right)$$

$$= \frac{5(15)^2}{n^3}\cdot\left(n^3 + \cdots\right) - \frac{7(15)^2}{n^2}\left(\frac{n^2 + \cdots}{2}\right)$$

$$= 5(15)^2\left(\frac{n^3 + \text{lower}}{n^3}\right) - \frac{7(15)^2}{2}\left(\frac{n^2 + \cdots}{n^2}\right)$$

$$\xrightarrow{n\to\infty} 15^2\left(5(1) - \frac{7}{2}(1)\right) = 15^2\left(\frac{10-7}{2}\right) = 225\left(\frac{3}{2}\right)$$

$$= \frac{675}{2} = \boxed{337.5}$$

$$\int_0^{15} (x^2 - 7x)\,dx = \frac{x^3}{3} - \frac{7x^2}{2}\Bigg]_0^{15}$$

By 2nd Fundamental Theorem of Calculus (FTC II)

$$= \frac{15^3}{3} - \frac{7(15)^2}{2} = 225\left[5 - \frac{7}{2}\right]$$

$$= 225\left[\frac{3}{2}\right] = \frac{675}{2} = 337.5$$

$f(x) = x^2 - 7x$

$x_k = a + k\Delta x = 0 + k\left(\frac{15-0}{5}\right) = 3k$

$\Delta x \sum_{k=1}^{5} f(x_k) = 3 \sum_{k=1}^{5} \left(x_k^2 - 7x_k\right) = 3 \sum_{k=1}^{5} (3k)^2 - 7(3k)]$

$= 3 \sum_{k=1}^{5} \left(9k^2 - 21k\right) = 9 \sum_{k=1}^{5} \left(3k^2 - 7k\right)$

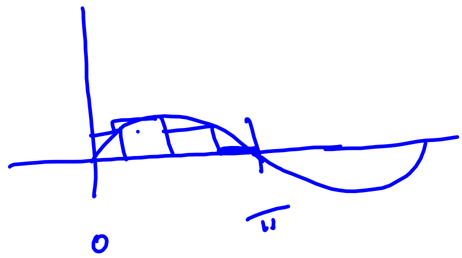
$= 9\left[3(1)^2 - 7(1) + 3(2)^2 - 7(2) + 3(3)^2 - 7(3) + 3(4)^2 - 7(4) + 3(5)^2 - 7(5)\right]$

$= 9\left[3 - 7 + 12 - 14 + 27 - 21 + 48 - 28 + 75 - 35\right]$

$= 9\left[165 - 105\right] = 9\left[60\right] = 540$ ✓

Do the same thing for $f(x) = \sin(x)$ on $[0, \pi]$

$n = 4$



$[a,b] = [0,\pi]$

$n = 4$

$\Delta x = \frac{b-a}{n} = \frac{\pi}{4}$

$x_k = a + k\Delta x = 0 + k\left(\frac{\pi}{4}\right) = \frac{\pi k}{4} = \frac{\pi}{4}k$

$$\text{Area} \approx \sum_{k=1}^{4} \sin(x_k)\Delta x = \Delta x \sum_{k=1}^{4} f(x_k) = \frac{\pi}{4}\sum_{k=1}^{4} \sin\left(\frac{\pi}{4}k\right)$$

$$= \frac{\pi}{4}\left[\sin\left(\frac{\pi}{4}\right) + \sin\left(\frac{2\pi}{4}\right) + \sin\left(\frac{3\pi}{4}\right) + \sin\left(\frac{4\pi}{4}\right)\right]$$

$$= \frac{\pi}{4}\left[\frac{\sqrt{2}}{2} + 1 + \frac{\sqrt{2}}{2} + 0\right] = \frac{\pi}{4}\left[\frac{2\sqrt{2}}{2}\right] = \boxed{\frac{\sqrt{2}\,\pi}{4}}$$

LEFT ENDPOINTS:

$x_1 = a \quad = a + (k-1)\Delta x$

$x_2 = a + \Delta x \quad = a + (k-1)\Delta x$

$x_3 = a + 2\Delta x \quad = a + (k-1)\Delta x$

$x_4 = a + 3\Delta x$

MIDPOINTS:

$x_1 = a + \frac{1}{2}\Delta x$

$x_2 = a + \frac{1}{2}\Delta x + \Delta x = a + \frac{3}{2}\Delta x$

$x_3 = a + \frac{3}{2}\Delta x + \Delta x = a + \frac{5}{2}\Delta x = a + \frac{2k-1}{2}\Delta x$

$x_4 = a + \frac{7}{2}\Delta x = a + \frac{2(4)-1}{2}\Delta x = \frac{2k-1}{2}\Delta x$

$x_k = a + \frac{2k-1}{2}\Delta x$