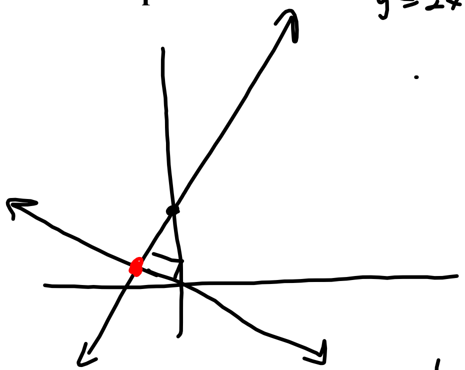
Find the point on the line $y = 2x + 3$ **that's closest to the origin.**



$y = -\frac{1}{2}x$ Find its intersection w/ $y = 2x+3$

$-\frac{1}{2}x = 2x + 3$

$-x = 4x + 6$

$-5x = 6$

$x = -\frac{6}{5}$ $\quad y = 2\left(-\frac{6}{5}\right) + 3 = \frac{-12+15}{5} = \frac{3}{5}$

$\boxed{\left(-\frac{6}{5}, \frac{3}{5}\right)}$

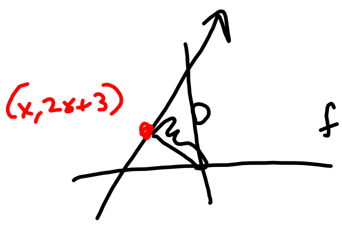
$D = \sqrt{(6/5)^2 + (3/5)^2}$

$= \sqrt{\frac{36+9}{25}} = \sqrt{\frac{45}{25}} = \sqrt{\frac{9}{5}} = \frac{3}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$

Now do it with calculus:

Minimize distance $= D = \sqrt{(x-0)^2 + (y-0)^2}$

$$= \sqrt{x^2 + (2x+3)^2}$$



$$f(x) = D^2 = x^2 + 4x^2 + 12x + 9$$

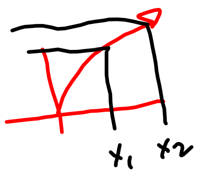
$$= 5x^2 + 12x + 9 \longrightarrow$$

$$f'(x) = 10x + 12 \overset{SET}{=} 0 \longrightarrow$$

$$x = -\frac{6}{5} \longrightarrow$$

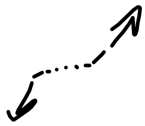
$$y = 2\left(-\frac{6}{5}\right) + 3 = \frac{3}{5}$$

$$(x,y) = \left(-\frac{6}{5}, \frac{3}{5}\right)$$



Square root function is increasing, so to minimize it, you minimize what's inside the square root.

$$R(x) = \frac{(x+1)^1(x-7)^1}{(x-2)^1} = \frac{x^2 + \cdots}{x - \cdots} = x + b + \frac{\sim}{x-2}$$



$\mathcal{D} = \mathbb{R} \setminus \{2\}$

No x−2 upstairs, so

$\boxed{x = 2 \text{ is V.A.}}$

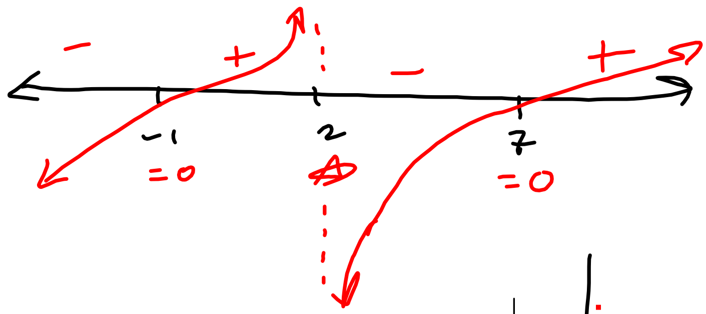
Zeros $x = -1, 7$

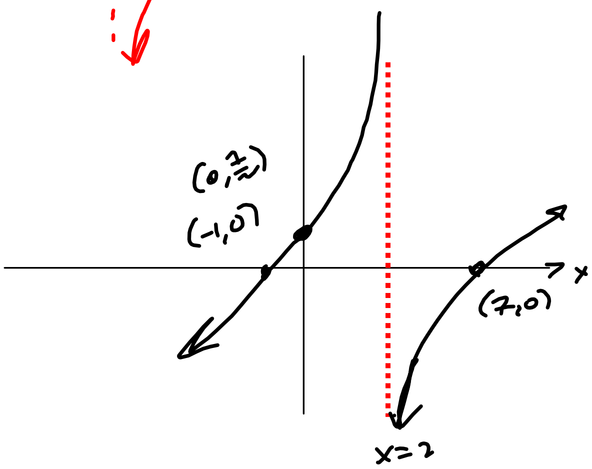
Cut points $x = -1, 7, 2$
$\qquad\qquad\quad =0 \;\; =0 \;\; A$

$\boxed{\text{y-int: } R(0) = \frac{(1)(-7)}{-2} = \frac{7}{2}}$

$\boxed{(0, \tfrac{7}{2})}$

$-1, \; 2, \; 7$
$=0 \;\; A \;\; =0$





Slant Asymptote (Oblique Asymptote)

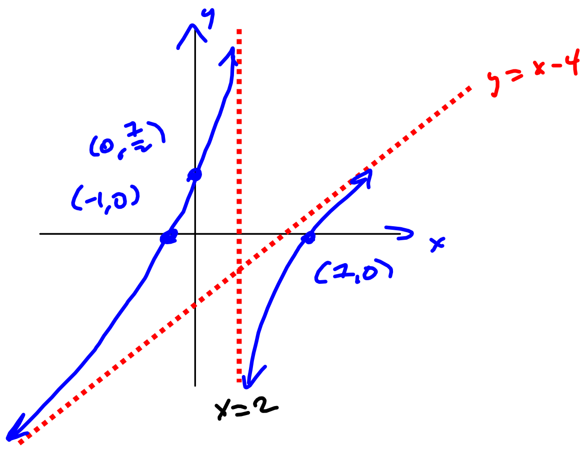
$(x+1)(x-7) = x^2 - 6x - 7$

Divide:

$x - 2 \,\overline{)\, x^2 - 6x - 7}$

$$\begin{array}{r|rrr} 2 & 1 & -6 & -7 \\ & & 2 & -8 \\ \hline & 1 & -4 & -15 \end{array}$$

$x - 4 - \frac{15}{x-2} = R(x)$



INFINITE LIMITS.

$$\lim_{x \to 2^-} R(x) = \infty$$

$$\lim_{x \to 2^+} R(x) = -\infty$$

See if there's any "wiggle" to it:

$$R(x) = \frac{x^2 - 6x - 7}{x - 2} \longrightarrow$$

$$R'(x) = \frac{(2x-6)(x-2) - (x^2 - 6x - 7)(1)}{(x-2)^2} = \frac{f'g - fg'}{g^2}$$

$$= \frac{2x^2 - 10x + 12 - x^2 + 6x + 7}{(x-2)^2} \qquad \nexists \text{ @ } x = 2$$

$$= \frac{x^2-4x+19}{(x-2)^2} \overset{\text{set}}{=} 0$$

$$x^2-4x+2^2-4+19 = (x-2)^2+15 \overset{\text{set}}{=} 0 \Rightarrow$$

$$\boxed{(x-2)^2=-15} \Rightarrow \text{No real sol'n.}$$

$$x-2 = \pm\sqrt{-15} = \pm i\sqrt{15} \notin \mathbb{R} \Rightarrow$$

No real zeros.

OR $b^2-4ac = 4^2-4(1)(19) < 0$ No real sol'n.

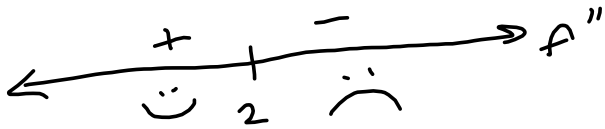
$$\boxed{\text{No local max/min}}$$

$$f' = \frac{x^2-4x+19}{(x-2)^2} \Rightarrow$$

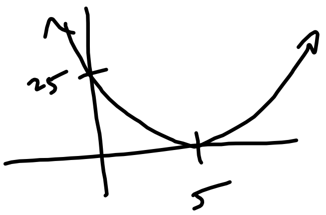
$$f''(x) = \frac{(2x-4)(x-2)^2-(x^2-4x+19)(2(x-2)')}{(x-2)^4}$$

$$= \frac{(x-2)\left[(2x-4)(x-2)-(2(x^2-4x+19))\right]}{(x-2)^4}$$

$$= \frac{2x^2-8x+8-2x^2+8x-38}{(x-2)^3} = \frac{-30}{(x-2)^3}$$



$f(x) = (x-5)^2$



According to the textbook definition, f is decreasing on $(-\infty, 5]$ & increasing on $[5, \infty)$, but then in chapter 3, they kind of say

dec: $(-\infty, 5)$

inc: $(5, \infty)$

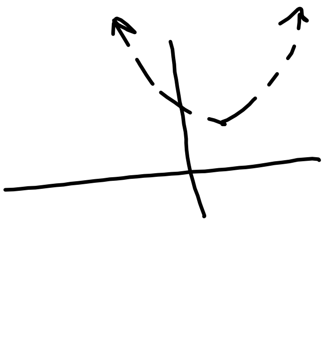
Intervals of concavity, I believe, is on the open interval.

A More General oblique asymptote

$$Q(x) = \frac{x^3+5x-3}{x+1}$$

$$\begin{array}{r|rrrr} -1 & 1 & 0 & 5 & -3 \\ & & -1 & 1 & -6 \\ \hline & 1 & -1 & 6 & -9 \end{array}$$

$$\Rightarrow Q(x) = x^2 - x + 6 - \frac{9}{x+1}$$



Quadratic Asymptote!