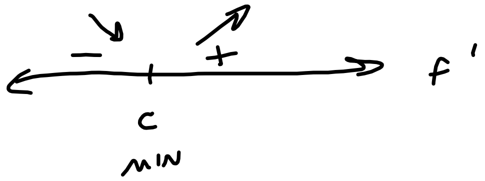
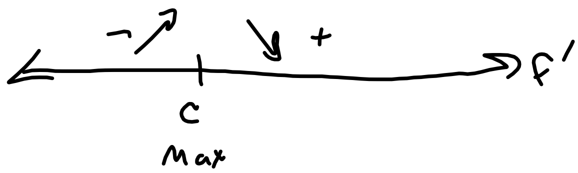
The first derivative test:

f '(c) = 0.

Then if f ' (x) < 0 for x < c and f ' (x) > 0 for x > c, f (c) is a minimum.



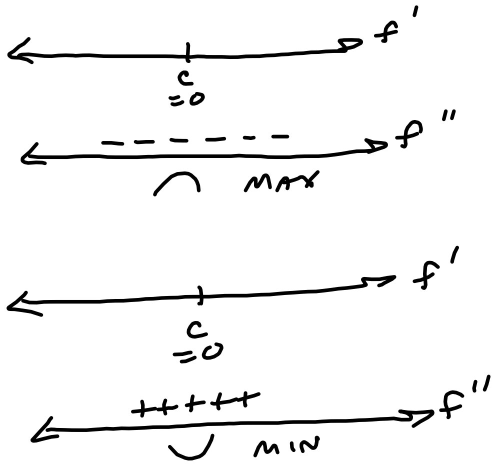
Then if f ' (x) > 0 for x < c and f ' (x) < 0 for x > c, f (c) is a maximum.



2nd-Derivative test. If f ' (c) = 0 and...

f '' (c) < 0, then f(c) is a maximum.

f '' (c) > 0, then f(c) is a minimum.



$f(x) = x\sin(x) \Longrightarrow$

$f'(x) = \sin(x) + x\cos(x)$

$f'g + fg'$

$f = x \quad g = \sin(x)$

Finding the zeros of f' is tricky

$\sin(x) + x\cos(x) = 0 \Longrightarrow$

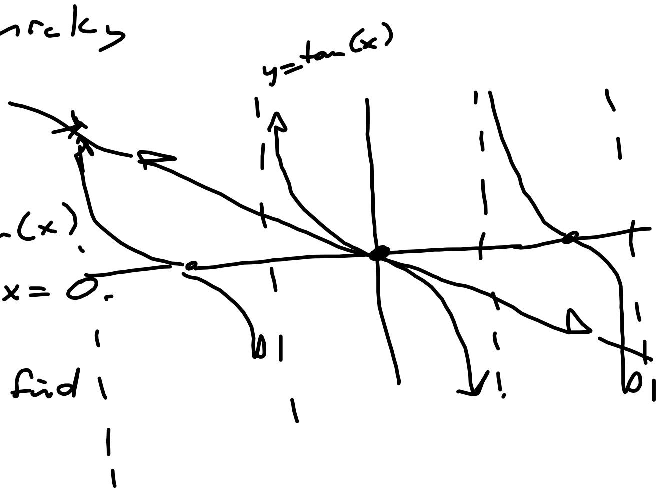
$x\cos(x) = -\sin(x)$

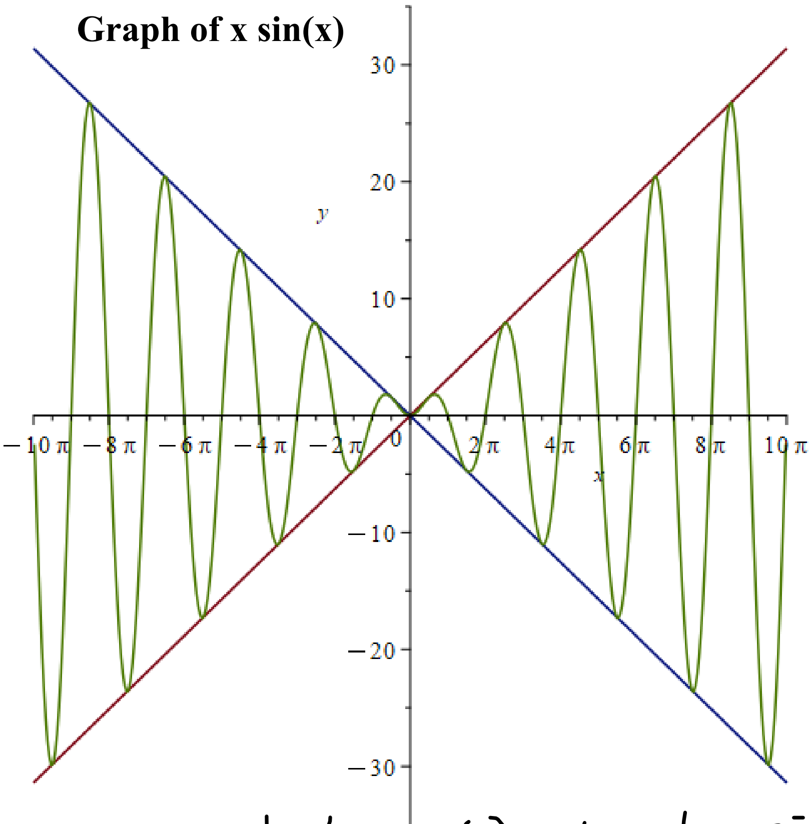
$x = \dfrac{-\sin(x)}{\cos(x)} = -\tan(x)$

Clearly, this is true for $x = 0$.

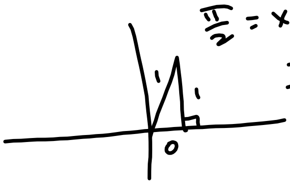
Any where else?

Lots of places. Hard to find





Makes sense that $x\sin(x) = x$ when $\sin(x) = 1$



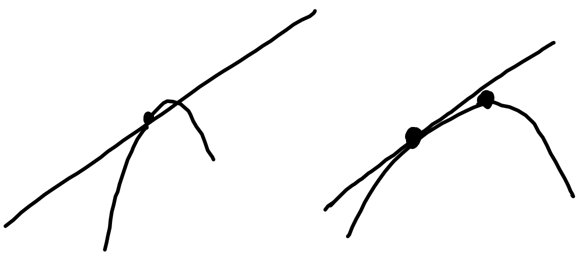
$\frac{\pi}{2} = x$

$\frac{\pi}{2} + 2n\pi$, $\forall n \in \mathbb{Z}$.

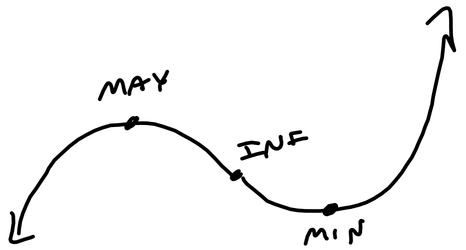
$\sin(x) = -1$ is when $x\sin(x)$ touches $y = -x$.

Going to have to come back to this.

We'll re-visit this example. But we can see how quickly things can spiral out of control.



Recall, the cubic from yesterday. It had 1 max, 1 min, and 1 inflection point.



How about a rational function?

$$R(x) = \frac{x-1}{x+2} \xrightarrow{|x| \to \infty} +1$$

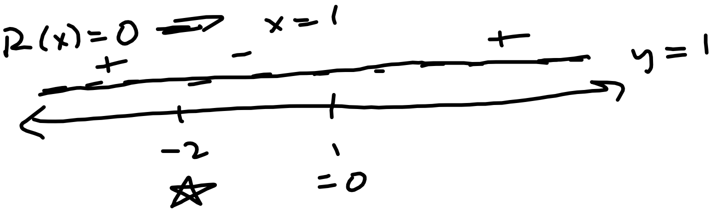
$$\mathcal{D} = \mathbb{R} \setminus \{-2\}$$

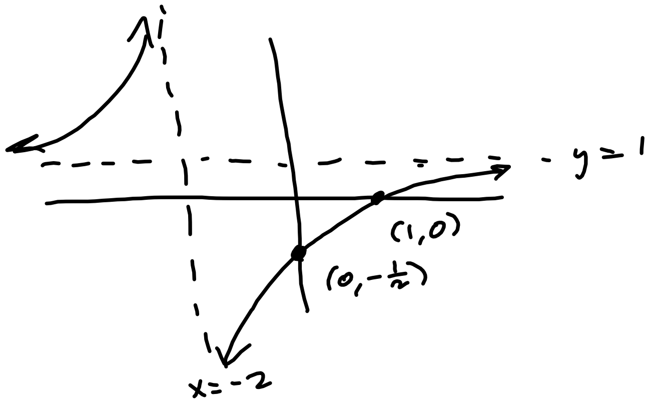
$x=-2$ is Vertical Asymptote (V.A.)

$R(0) = -\frac{1}{2}$

$\rightsquigarrow (0, -\frac{1}{2})$

$R(x) = 0 \Rightarrow x = 1$



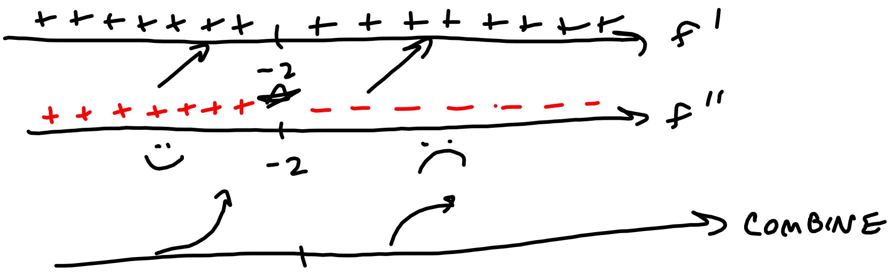


$$R'(x) = \frac{1(x+2) - (x-1)(1)}{(x+2)^2} = \frac{x+2-x+1}{(x+2)^2} = \frac{3}{(x+2)^2}$$ always positive on its domain.

$x=-2$

$$R''(x) = \frac{d}{dx}\left[\frac{3}{(x+2)^2}\right] = \frac{d}{dx}\left[3(x+2)^{-2}\right] = -6(x+2)^{-3}$$

$$= \frac{-6}{(x+2)^3}$$ cut/key points: $x=-2$



This says that the college-algebra graph is good for calculus, as well!

Let's do something the College Algebra kids can't do.

$$R(x)=\frac{10x^2-59x+84}{3x^2-7x-10}$$

END BEHAVIOR:

$$\frac{\cancel{x^2}\left(10-\frac{59}{x}+\frac{84}{x^2}\right)}{\cancel{x^2}\left(3-\frac{7}{x}-\frac{10}{x^2}\right)}$$

$\frac{59}{x}\to 0$, $\frac{84}{x^2}\to 0$, $\frac{7}{x}\to 0$, $\frac{10}{x^2}\to 0$

$x\to\pm\infty \longrightarrow \frac{10}{3}$

$y=\frac{10}{3}$ is Horizontal Asymptote

FACTOR TOP & BOTTOM

You can always cheat factoring quadratics with the Quadratic Formula.

$10x^2-59x+84=0$

$a=10,\ b=-59,\ c=84$

$\Rightarrow b^2-4ac = 59^2-4(10)(84)$

$= 3481-3360 = 121 = 11^2 \rightsquigarrow \sqrt{11^2}=11$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{59\pm 11}{20}$ → $\frac{70}{20}=\frac{7}{2}$, $\frac{48}{20}=\frac{12}{5}$

Now, reverse-engineer the factorization.

$\Rightarrow 10x^2-59x+84 = 10\left(x-\frac{7}{2}\right)\left(x-\frac{12}{5}\right)$

$= 5\cdot 2\left(x-7/2\right)\left(x-\frac{12}{5}\right)$

$=(2x-7)(5x-12)$!

$3x^2 - 7x - 10$

$ac = -30$
want factors of -30 that add together to give -7.

$3x^2 - 10x + 3x - 10$

$= x(3x-10) + 1(3x-10)$

$= (3x-10)(x+1)$

$$R(x) = \frac{(2x-7)^1(5x-12)^1}{(3x-10)^1(x+1)^1}$$

$\mathcal{D} = \mathbb{R} \setminus \{ \frac{10}{3}, -1 \}$

V.A.: $x = \frac{10}{3}$, $x = -1$

3.5 2.4

$R(x) = 0 \Longrightarrow (2x-7)(5x-12) = 0$ i.e., $x = \frac{7}{2}, \frac{12}{5}$

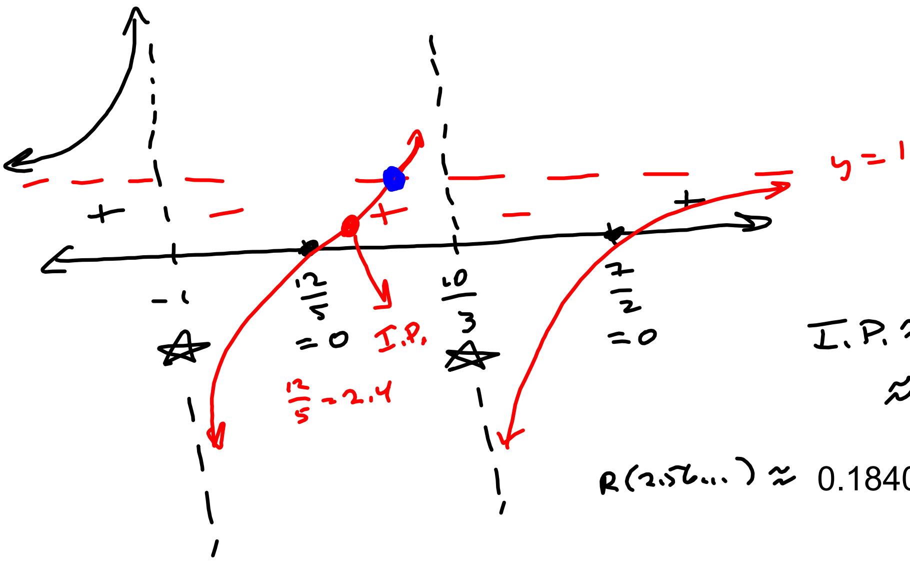
$= \frac{12}{5}, \frac{7}{2}$

2.4 3.5

Important / key / cut points:

$-1, \frac{10}{3} = 3.\overline{3}, \frac{12}{5}, \frac{7}{2}$

$-1, \frac{12}{5}, \frac{10}{3}, \frac{7}{2}$ from left to right



I.P. ≈ (2.56, R(2.56))

≈ (2.56, 0.18)

R(2.56...) ≈ 0.1840555093

Note: $y = \frac{10}{3}$ intersects R(x) here: x = 352/107 ≈ 3.289719626

For some reason, I thought the intersection should be off to the right of x = 7/2, but it's slightly to the left of x = 10/3. That's not a problem.

$$R(x) = \frac{10x^2 - 59x + 84}{3x^2 - 7x - 10} \qquad \left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

We've done all we could with Algebra.

$$R'(x) = \frac{(20x-59)(3x^2-7x-10) - (10x^2-59x+84)(6x-7)}{(3x^2-7x-10)^2}$$

$$= \frac{60x^3 - 140x^2 - 200x - 177x^2 + \cancel{413x} - 590 - [60x^3 - 70x^2 + 354x^2 + \cancel{413x} + 504x - 588]}{((3x-10)(x+1))^2}$$

$84 \cdot 7 = 588 \quad 59 \cdot 3 = 177 \quad 59 \cdot 7 = 413 \quad 59 \cdot 6 = 354 \quad 84 \cdot 6 = 504$

$$= \frac{-70x^2 - 200x - 177x^2 - 590 - 354x^2 - 504x + 588}{((3x-10)(x+1))^2}$$

$$= \frac{-601x^2 - 704x - 2}{(\quad)^2} \qquad -247 - 354 = -601$$

Nope. You screwed something up by rushing and not using a calculator, appropriately, Steve.

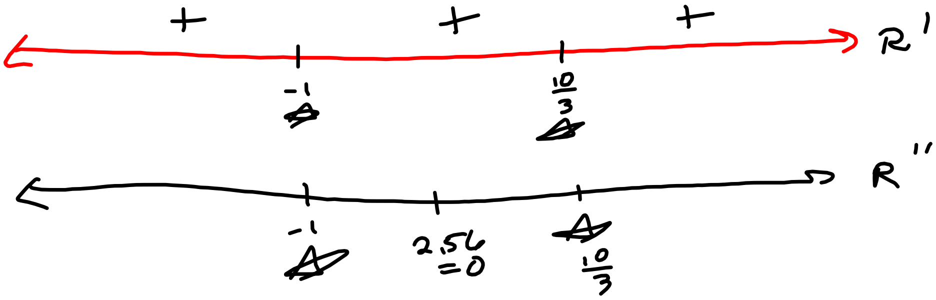
$$R'(x) = \frac{107x^2 - 704x + 1178}{(3x^2 - 7x - 10)^2}, \text{ by CAS.}$$

$b^2 - 4ac < 0 \rightarrow R'(x) \neq 0$ so no local max/min.

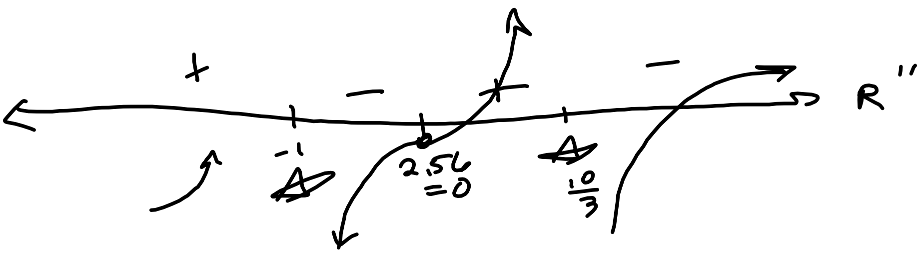
$$R''(x) = -\frac{6(107x^3 - 1056x^2 + 3534x - 3922)}{(3x^2 - 7x - 10)^3}. \quad \underline{\text{SET}}\ 0$$

$\rightarrow$ $x \approx 2.561732534$, by solving $107x^3 - 1056x^2 + 3534x - 3922 = 0$

SIGN PATTERN FOR R' & R''



$$R'(x) = \frac{107\,x^2 - 704\,x + 1178}{\left(3\,x^2 - 7\,x - 10\right)^2} = \frac{\sim\sim}{(3x-10)^2(x+1)^2}$$

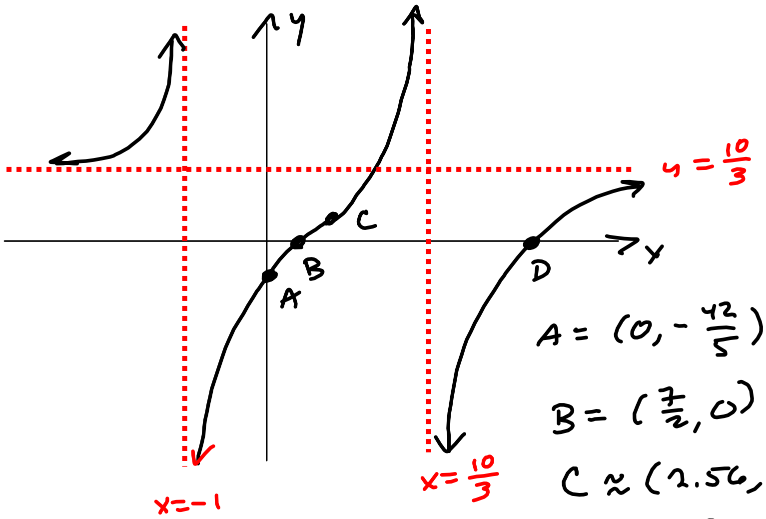


$$R''(x) = -\frac{6\left(107\,x^3 - 1056\,x^2 + 3534\,x - 3922\right)}{\left(3\,x^2 - 7\,x - 10\right)^3} = \frac{\sim\sim}{(3x-10)^3(x+1)^3}$$

$(x-2.56..)^1$ (Imaginary stuff)

$x \approx$ 2.561732534

Now, do the graph!



$A = \left(0, -\frac{42}{5}\right)$ y-int

$B = \left(\frac{7}{2}, 0\right)$ x-int

$C \approx (2.56, 0.18)$ I.P.

$D = \left(\frac{12}{5}, 0\right)$ x-int.

$$R(x) = \frac{(2x-7)^1(5x-12)^1}{(3x-10)^1(x+1)^1}$$

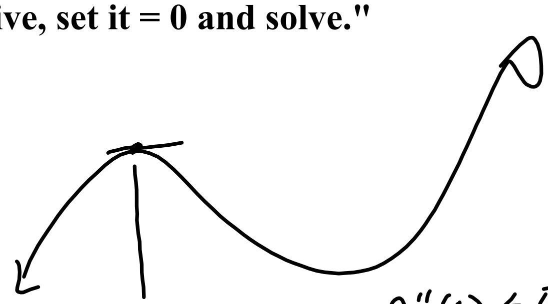
We'll do more graphs.

But first, a word or two on "Optimization"

Maximize/Minimize a function.

Calculus is great for this!

"Take the derivative, set it = 0 and solve."



c $f'(c)=0,\ f''(c)<0 \Rightarrow$ MAX!

(Local)

Put in a reasonable amount of time and ask.

Take-Home Test 3 is coming up. I'll be handing it out this week.

It'll contain two or three graphing questions.

One easy. One hard. Maybe one intermediate.

Next time, some WebAssign Homework graphs. 3.5

After that, Newton's Method.