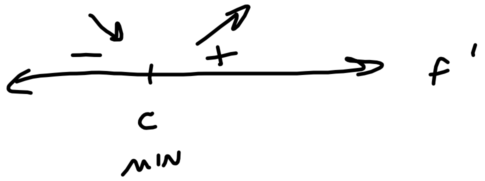
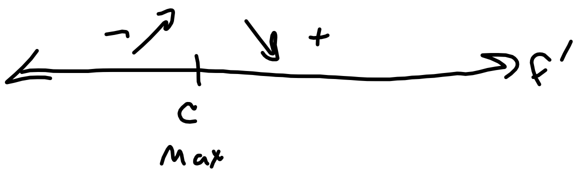
The first derivative test:

f '(c) = 0.

Then if f ' (x) < 0 for x < c and f ' (x) > 0 for x > c, f (c) is a minimum.



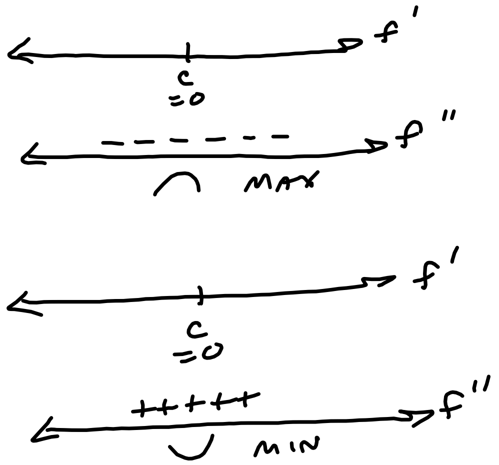
Then if f ' (x) > 0 for x < c and f ' (x) < 0 for x > c, f (c) is a maximum.



2nd-Derivative test. If f ' (c) = 0 and...

f '' (c) < 0, then f(c) is a maximum.

f '' (c) > 0, then f(c) is a minimum.



$f(x) = x\sin(x) \implies$

$f'(x) = \sin(x) + x\cos(x)$

$f'g + fg'$

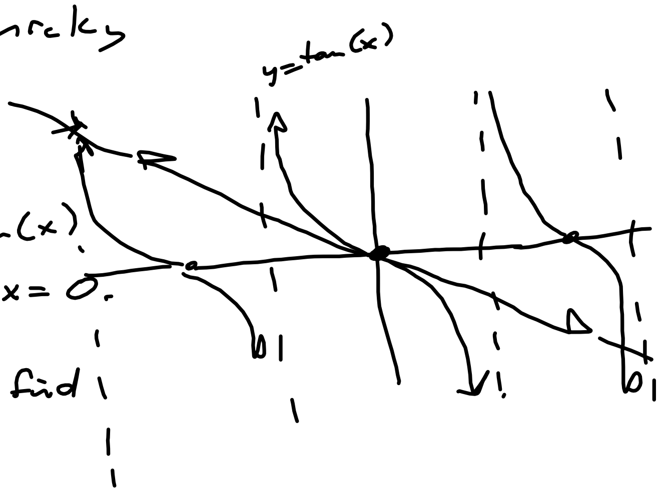
$f = x \qquad g = \sin(x)$

Finding the zeros of f' is tricky

$\sin(x) + x\cos(x) = 0 \implies$

$x\cos(x) = -\sin(x)$

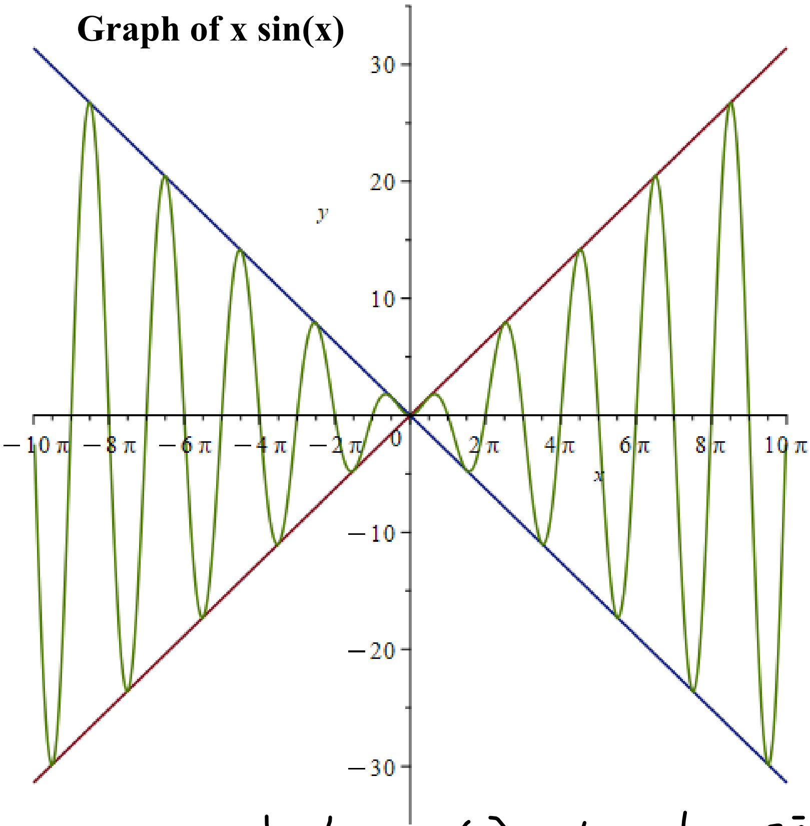
$x = \dfrac{-\sin(x)}{\cos(x)} = -\tan(x)$



Clearly, this is true for $x = 0$.

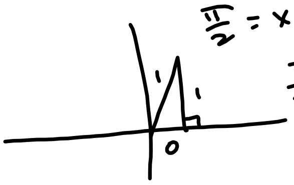
Any where else?

Lots of places. Hard to find



Makes sense that $x \sin(x) = x$ when $\sin(x) = 1$

$\frac{\pi}{2} = x$



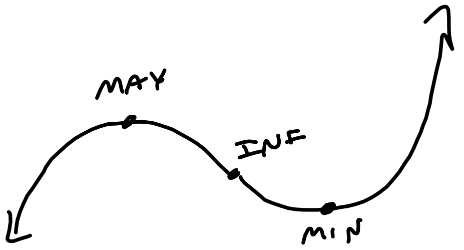
$\frac{\pi}{2} + 2n\pi$, $\forall n \in \mathbb{Z}$.

$\sin(x) = -1$ is when $x \sin(x)$ touches $y = -x$.

Going to have to come back to this.

We'll re-visit this example. But we can see how quickly things can spiral out of control.

Recall, the cubic from yesterday. It had 1 max, 1 min, and 1 inflection point.



How about a rational function?

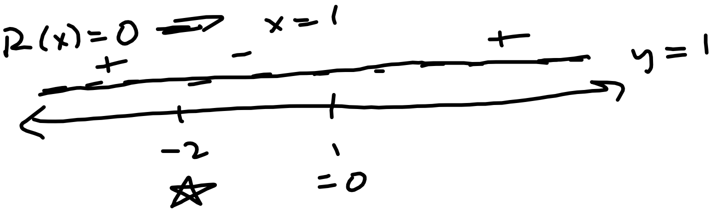
$R(x) = \frac{x-1}{x+2} \xrightarrow{|x| \to \infty} +1$

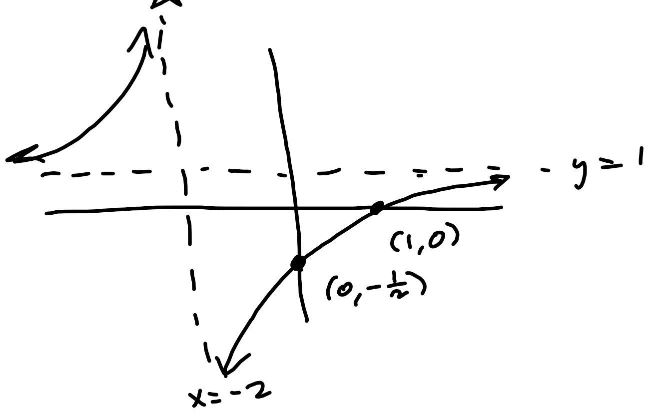
$\mathcal{D} = \mathbb{R} \setminus \{-2\}$

$x=-2$ is Vertical Asymptote (V.A.)

$R(0) = -\frac{1}{2} \rightsquigarrow (0, -\frac{1}{2})$

$R(x) = 0 \Rightarrow x = 1$



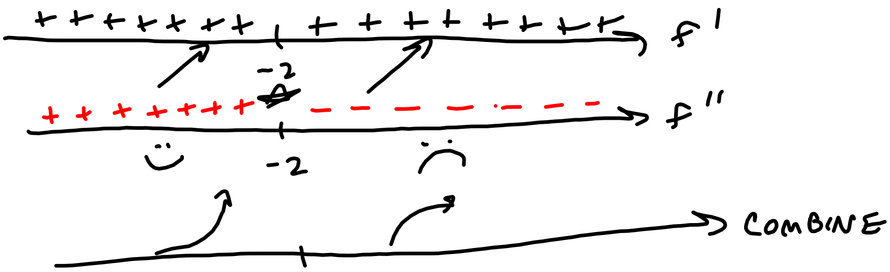


$$R'(x) = \frac{1(x+2)-(x-1)(1)}{(x+2)^2} = \frac{x+2-x+1}{(x+2)^2} = \frac{3}{(x+2)^2}$$ always positive on its domain.

$x = -2$ ☆

$$R''(x) = \frac{d}{dx}\left[\frac{3}{(x+2)^2}\right] = \frac{d}{dx}\left[3(x+2)^{-2}\right] = -6(x+2)^{-3}$$

$$= \frac{-6}{(x+2)^3}$$ cut/key points: $x=-2$ ☆



This says that the college-algebra graph is good for calculus, as well!

Let's do something the College Algebra kids can't do.

$$R(x)=\frac{10x^2-59x+84}{3x^2-7x-10}$$

END BEHAVIOR:

$$\frac{\cancel{x^2}\left(10-\frac{59}{x}+\frac{84}{x^2}\right)}{\cancel{x^2}\left(3-\frac{7}{x}-\frac{10}{x^2}\right)}$$

$\frac{59}{x} \to 0$, $\frac{84}{x^2} \to 0$, $\frac{7}{x} \to 0$, $\frac{10}{x^2} \to 0$

$x \to \pm\infty \longrightarrow \frac{10}{3}$

$y=\frac{10}{3}$ is Horizontal Asymptote

FACTOR TOP & BOTTOM

You can always cheat factoring quadratics with the Quadratic Formula.

$10x^2-59x+84=0$

$a=10,\ b=-59,\ c=84$

$\Rightarrow b^2-4ac = 59^2-4(10)(84)$

$= 3481-3360 = 121 = 11^2 \rightsquigarrow \sqrt{11^2}=11$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{59\pm 11}{20} \begin{cases} \frac{70}{20}=\frac{7}{2} \\ \frac{48}{20}=\frac{12}{5}\end{cases}$$

Now, reverse-engineer the factorization.

$\Rightarrow 10x^2-59x+84 = 10\left(x-\frac{7}{2}\right)\left(x-\frac{12}{5}\right)$

$= 5\cdot 2\left(x-\frac{7}{2}\right)\left(x-\frac{12}{5}\right)$

$= (2x-7)(5x-12)$!

$3x^2 - 7x - 10$ $\quad ac = -30$

want factors of -30 that add together to give -7.

$3x^2 - 10x + 3x - 10$

$= x(3x-10) + 1(3x-10)$

$= (3x-10)(x+1)$

$$R(x) = \frac{(2x-7)^1(5x-12)^1}{(3x-10)^1(x+1)^1}$$

$\mathcal{D} = \mathbb{R} \setminus \{\frac{10}{3}, -1\}$

V.A.: $x = \frac{10}{3}$, $x = -1$

$$3.5 \quad 2.4$$

$R(x) = 0 \Longrightarrow (2x-7)(5x-12) = 0$ i.e., $x = \frac{7}{2}, \frac{12}{5}$

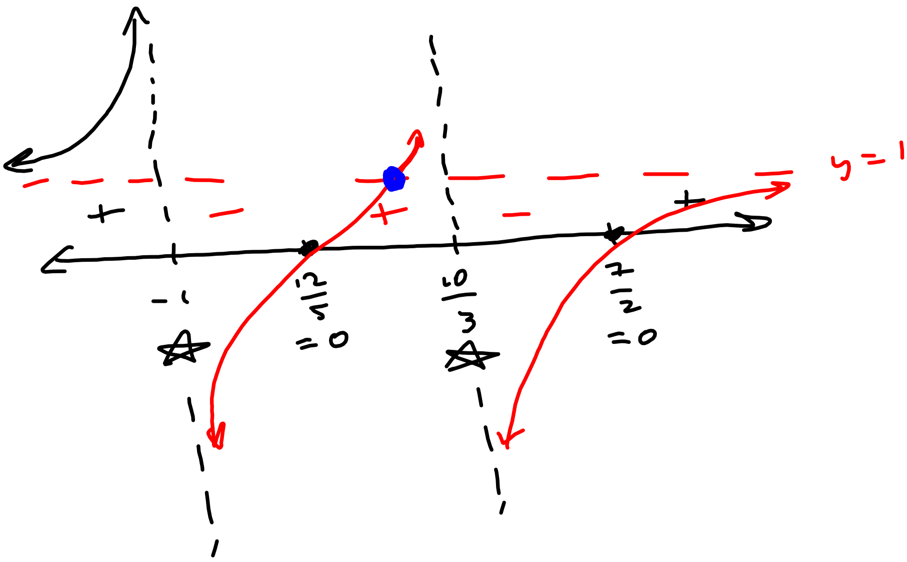
$= \frac{12}{5}, \frac{7}{2}$

$2.4 \quad 3.5$

Important / key / cut points:

$-1, \frac{10}{3} = 3.\overline{3}, \frac{12}{5}, \frac{7}{2}$

$-1, \frac{12}{5}, \frac{10}{3}, \frac{7}{2}$ from left to right.



Note: $y = \frac{10}{3}$ intersects $R(x)$ here: $x = 352/107 \approx 3.289719626$

For some reason, I thought the intersection should be off to the right of $x = 7/2$, but it's slightly to the left of $x = 10/3$. That's not a problem.