1. (10 pts) $\lim_{x\to 3}\frac{x^2-9}{x^3-27}$: $\lim_{x\to 3}\frac{(x-3)(x+3)}{(x-3)(x^2+3x+9)}$

$= \lim_{x\to 3}\frac{x+3}{x^2+3x+9} = \frac{3+3}{3^2+3\cdot 3+9} = \frac{6}{27} = \boxed{\frac{2}{9}}$

2a. (5 pts) $\lim_{x\to -3}\frac{2x^2+x-15}{x^2+5x+6}$: $\frac{2x^2+x-15}{x^2+5x+6} = \frac{(2x-5)(x+3)}{(x+2)(x+3)}$

$= \frac{2x-5}{x+2} \xrightarrow{x\to -3} \frac{2(-3)-5}{-3+2} = \frac{-11}{-1} = \boxed{11}$

$(x\neq -3)$

2b. (5 pts) $\lim_{x\to -3}\frac{2x^2-x-15}{x^2+5x+6}$: $\frac{2x^2-x-15}{x^2+5x+6} = \frac{(2x+5)(x-3)}{(x+3)(x+2)}$

Nothing cancels.
$x \to -3$ is impossible.

$2x^2-x-15$:
$b^2-4ac = 1-4(2)(-15)$
$= 1+120 = 121$ FACTORS $ac = -30 =$ magic

middle term: $-1 = -2+1$ -2
$= -6+5$ -30 ✓
$2x^2-6x+5x-15$
$= 2x(x-3)+5(x-3)$
$= (x-3)(2x+5)$

Ray's Office: HOR 103

Testing Room: Suite 107

Open hours: 7 am - 8 pm

RocketBook for making PDFs out for cheap.

Free "Scannable" app for iPhone (Evernote Brand)

I've seen good results from Camscanner.

GeniusScan app

Notability app. Save on iPad and just upload to D2L.

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

$$(fg)' = f'g + fg'$$

General Power Rule:

$$\frac{d}{dx}\left[f(x)^n\right] = n\,f(x)^{n-1} \cdot f'(x)$$

$$\frac{d}{dx}\left[(x^2-5x)^7\right] = 7(x^2-5x)(2x-5)$$

Chain Rule

$$\frac{d}{dx}\left[f(g(x))\right] = \frac{df}{dg} \cdot \frac{dg}{dx} = f'(g)\,g'(x)$$

3. $\lim_{x\to 5}(2x-7)=3$

Proof

Let $\varepsilon>0$ be given. Define $\delta=\frac{\varepsilon}{2}$.

Then $0<|x-5|<\delta \implies |f(x)-3| = |2x-7-3| = |2x-10| = 2|x-5| < 2\delta = 2\left(\frac{\varepsilon}{2}\right)=\varepsilon$ ∎

4. (10 pts) $f(x)=x^2+5x+6 \implies$

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^2+5(x+h)+6-(x^2+5x+6)}{h}$$

$$= \frac{x^2+2xh+h^2+5x+5h+6-x^2-5x-6}{h} =$$

$$= \frac{2xh+h^2+5h}{h} = 2x+h+5 \; (h\neq 0) \xrightarrow{h\to 0} \boxed{2x+5=f'(x)}$$

$$= \frac{h(2x+h+5)}{h}$$

5. a. (5 pts.) $y = x^2 + 5x + \frac{6}{x^2}$ $= x^2 + 5x + 6x^{-2} \longrightarrow$

$\boxed{y' = 2x + 5 - 12x^{-3}}$

$(fg)' = f'g + fg'$

b. (5 pts.) $y = (x^2+5x)(7x-1)$ $\longrightarrow$

$\boxed{y' = (2x+5)(7x-1) + (x^2+5x)(7)}$

$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$

c. (5 pts.) $y = \frac{x^2+5x}{7x-1}$ $\longrightarrow$

$\boxed{y' = \frac{(2x+5)(7x-1) - (x^2+5x)(7)}{(7x-1)^2}}$

$f = x^2+5x \Rightarrow f' = 2x+5$

$g = 7x-1 \Rightarrow g' = 7$

$f = (x^2+5x)^3 \to f' = 3(x^2+5x)^2(2x+5)$

$g = (7x-1)^5 \Rightarrow g' = 5(7x-1)^4(7)$

d. (5 pts.) $y = (x^2+5x)^3(7x-1)^5$ $\longrightarrow$

$\boxed{y' = 3(x^2+5x)^2(2x+5)(7x-1)^5 + (x^2+5x)^3(5(7x-1)^4)(7)}$

$= (x^2+5x)^2(7x-1)^4\left[3(2x+5)(7x-1) + 35(x^2+5x)\right]$ NATE MARTINEZ

$= (x^2+5x)^2(7x-1)^4\left[3(14x^2+33x-5) + 35x^2 + 175x\right]$

$= (x^2+5x)^2(7x-1)^4\left[42x^2+99x-15+35x^2+175x\right]$

$= (x^2+5x)^2(7x-1)^4\left[77x^2+274x-15\right]$

e. (5 pts.) $y = \cot(\sec(x^2-5))$ $\longrightarrow$

$\boxed{y' = -\csc^2(\sec(x^2-5))(\sec(x^2-5)\tan(x^2-5))(2x)}$

$f(g(h(x))) = K(x)$

$f(g(h))' = f'(g(h))\,g'(h)\,h'(x)$

$\frac{d}{dx}\left[f(g(h(x)))\right] = \frac{df}{dg}\cdot\frac{dg}{dh}\cdot\frac{dh}{dx}$

$f = \cot(g)$ $\quad \frac{df}{dg} = -\csc^2(g)$

$g = \sec(h)$ $\quad \frac{dg}{dh} = \sec(h)\tan(h)$

$h = x^2-5$ $\quad \frac{dh}{dx} = 2x$

$\longrightarrow K'(x) = -\csc^2(g)\sec(h)\tan(h)\cdot 2x$

$= -\csc^2(\sec(x^2-5))\sec(x^2-5)\tan(x^2-5)\cdot 2x$

6. (10 pts) Find an equation of the tangent line to $f(x) = \sin(x)$ at $x = \frac{\pi}{3}$. Then sketch the graph of this situation, with the function and its tangent line, together on the same set of axes.

$f(x) = \sin(x), \quad x_1 = \frac{\pi}{3}. \quad \longrightarrow$

$f(x_1) = y_1 = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$, &

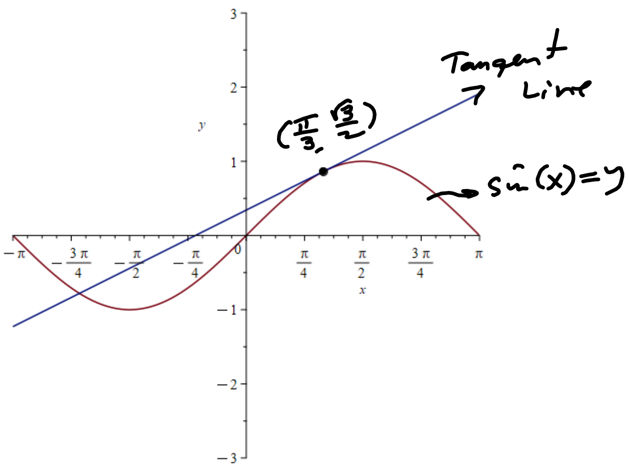
$f'(x_1) = \cos(x_1) = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$

Tangent Line is

$y = f'(x_1)(x - x_1) + f(x_1)$

$$\boxed{y = \frac{1}{2}\left(x - \frac{\pi}{3}\right) + \frac{\sqrt{3}}{2}}$$

$y = m(x - x_1) + y_1$



7. (5 pts) Use your result from the previous problem to approximate $\sin(65^\circ)$

$\frac{\pi}{3} = 60^\circ$

$65^\circ = 60^\circ + 5^\circ = \frac{\pi}{3} + (5^\circ)\left(\frac{\pi}{180^\circ}\right) = \frac{\pi}{3} + \frac{\pi}{36} = x_2$

$\Delta x = \frac{\pi}{36} = x - x_1$

Δx

$y = L_{\frac{\pi}{3}}(x) = \frac{1}{2}\left(x - \frac{\pi}{3}\right) + \frac{\sqrt{3}}{2} \Longrightarrow$

$L_{\frac{\pi}{3}}(x) = \frac{1}{2}\left(\underbrace{\frac{\pi}{3} + \frac{\pi}{36} - \frac{\pi}{3}}_{\Delta x}\right) + \frac{\sqrt{3}}{2}$

$= \frac{1}{2}\left(\frac{\pi}{36}\right) + \frac{\sqrt{3}}{2}$

$= \boxed{\frac{\pi}{72} + \frac{\sqrt{3}}{2} \approx \sin(65^\circ)}$ 0.9096586353

Actual: $\sin(65^\circ) \approx$ 0.9063077871

$y = L_{\frac{\pi}{3}}(x) = \frac{1}{2}\left(x - \frac{\pi}{3}\right) + \frac{\sqrt{3}}{2} \Longrightarrow$

$L_{\frac{\pi}{3}}(65^\circ) = L_{\frac{\pi}{3}}\left(\frac{65\pi}{180}\right) = \frac{1}{2}\left(\frac{13\pi}{36} - \frac{\pi}{3}\right) + \frac{\sqrt{3}}{2}$

radians essential

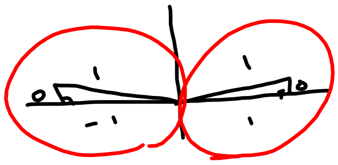
$= \frac{1}{2}\left(\frac{13\pi - 12\pi}{36}\right) + \frac{\sqrt{3}}{2}$

$= \frac{1}{2}\left(\frac{\pi}{36}\right) + \frac{\sqrt{3}}{2}$

$= \boxed{\frac{\pi}{72} + \frac{\sqrt{3}}{2} \approx \sin(65^\circ)}$

8. (10 pts) Find all values of x such that $f(x)=1+2\cos(x)$ has a horizontal tangent.

$f'(x) = -2\sin(x) \overset{\text{SET}}{=} 0$



$\underbrace{x = 0 + 2\pi n \quad \text{OR} \quad x = \pi + 2\pi n}_{\text{combine!}}$

$x = \pi n.$

Solution-set answer.

$$\boxed{\therefore \{x \mid f'(x)=0\} = \{\pi n \mid n \in \mathbb{Z}\}.}$$

9. (10 pts) Find $\frac{dy}{dx}$, given that $\sec x + \sin y = 2xy - 3x^2y^2$

$\sec(x)\tan(x) + \cos(y)y' = 2y + 2xy' - 6xy^2 - 6x^2yy'$

$(\cos(y) - 2x + 6x^2y)y' = 2y - 6xy^2 - \sec(x)\tan(x)$

$$\boxed{y' = \frac{2y - 6xy^2 - \sec(x)\tan(x)}{\cos(y) - 2x + 6x^2y}}$$

B1 (5 pts) Find the derivative of $f(x)=\frac{1}{\sqrt{x}}$, by the definition of the derivative.

$$\frac{f(x+h)-f(x)}{h} = \frac{1}{h}\left[f(x+h)-f(x)\right]$$

$$= \frac{1}{h}\left[\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}\right] = \frac{1}{h}\left[\frac{1}{\sqrt{x+h}}\cdot\frac{\sqrt{x}}{\sqrt{x}} - \frac{1}{\sqrt{x}}\cdot\frac{\sqrt{x+h}}{\sqrt{x+h}}\right]$$

$$= \frac{1}{h}\left[\frac{\sqrt{x}-\sqrt{x+h}}{\sqrt{x}\sqrt{x+h}}\right] = \frac{1}{h}\left[\frac{(\sqrt{x}-\sqrt{x+h})(\sqrt{x}+\sqrt{x+h})}{\sqrt{x}\sqrt{x+h}(\sqrt{x}+\sqrt{x+h})}\right]$$

$$= \frac{1}{h}\left[\frac{x-(x+h)}{\sqrt{x}\sqrt{x+h}(\sqrt{x}+\sqrt{x+h})}\right] = \frac{1}{h}\left[\frac{-h}{\sqrt{x}\sqrt{x+h}(\sqrt{x}+\sqrt{x+h})}\right]$$

$$= \frac{-1}{\sqrt{x}\sqrt{x+h}(\sqrt{x}+\sqrt{x+h})} \xrightarrow{h\to 0} \frac{-1}{x(2\sqrt{x})} = \boxed{\frac{-1}{2x\sqrt{x}} = f'(x)} = -\frac{1}{2x^{3/2}}$$

$(h \neq 0)$

$$f(x) = \frac{1}{\sqrt{x}} = \frac{1}{x^{\frac{1}{2}}} = x^{-\frac{1}{2}} \longrightarrow$$

$$f'(x) = -\frac{1}{2}x^{-\frac{1}{2}-1} = -\frac{1}{2}x^{-\frac{1}{2}-\frac{2}{2}} = -\frac{1}{2}x^{-\frac{3}{2}}$$

$$= \frac{-1}{2x^{3/2}} = \frac{-1}{2\sqrt{x^3}} = \frac{-1}{2\sqrt{x^2\cdot x}}$$

$$= \frac{-1}{2\sqrt{x^2}\sqrt{x}} = \frac{-1}{2|x|\sqrt{x}}$$. But

we already KNOW $x>0$,

b/c $\mathcal{D}\left(\frac{1}{\sqrt{x}}\right) = (0,\infty)$

so $|x| = x$, here & we

have $f'(x) = \frac{-1}{2x\sqrt{x}}$

B2 (5 pts) Prove that $\lim_{x\to 3}\left(x^2+5x+2\right)=26$

Scratch: $|x^2+5x+2-26| = |x^2+5x-24| = |x+8||x-3|$

$\delta \le 1 \Rightarrow 2 < x < 4 \Rightarrow 10 < x+8 < 12 \Rightarrow |x+8| < 12$

Need a bound on $|x+8|$ when x is "close" to $x=3$.

$< \delta$

Proof

Let $\varepsilon > 0$ be given. Define $\delta = \min\{1, \frac{\varepsilon}{12}\}$.

Then $0 < |x-3| < \delta \Rightarrow \underbrace{|x^2+5x+2-26|}_{|f(x)-26| =} = |x^2+5x-24|$

$= |x+8||x-3| < 12|x-3| < 12\delta \le 12 \cdot \frac{\varepsilon}{12} = \varepsilon$ ■

$\delta \le 1 \Rightarrow |x-3| < \delta \le 1$, i.e., $|x-3| < 1$

$$\Rightarrow \begin{array}{rcccl} -1 & < & x-3 & < & 1 \\ +11 & = & +11 & = & +11 \\ \hline 10 & < & x+8 & < & 12 \end{array}$$

i.e., $\underbrace{|x+8| < 12}$!

Define $\delta = \min\{1, \frac{\varepsilon}{12}\}$