43. A television camera is positioned 4000 ft from the base of a rocket launching pad. The angle of elevation of the camera has to change at the correct rate in order to keep the rocket in sight. Also, the mechanism for focusing the camera has to take into account the increasing distance from the camera to the rising rocket. Let's assume the rocket rises vertically and its speed is 600 ft/s when it has risen 3000 ft.

(a) How fast is the distance from the television camera to the rocket changing at that moment? $\left.\frac{dr}{dt}\right|_{x=3000} = ?$

(b) If the television camera is always kept aimed at the rocket, how fast is the camera's angle of elevation changing at that same moment? $\left.\frac{d\theta}{dt}\right|_{x=3000} = ?$

Let r = the distance from camera to the rocket (ft)

t = time, in seconds

x = height of rocket (ft)

θ = angle of elevation of the camera.

Given $\left.\frac{dx}{dt}\right|_{x=3000} = 600$ ft/s

(a) We find $\left.\frac{dr}{dt}\right|_{x=3000}$

$$r^2 = x^2 + 4000^2 \longrightarrow$$

$$r = \sqrt{x^2 + 4000^2}, \text{ b/c } r > 0.$$

$r = (x^2 + 4000^2)^{\frac{1}{2}} \Longrightarrow$

$\frac{dr}{dt} = \frac{1}{2}(x^2+4000^2)^{-\frac{1}{2}}\left(2x\frac{dx}{dt}\right)$ by chain rule on

$x = x(t)$.

$\Longrightarrow \left.\frac{dr}{dt}\right|_{x=3000} = \frac{1}{2}(3000^2+4000^2)^{-\frac{1}{2}}(2(3000)(600))$

$= \frac{1}{2}\frac{1}{\sqrt{25(10^6)}}(2)(3000)(600)$

$= \frac{1}{5\cdot 10^3}(3)(6)\cdot 10^5$

$= \frac{18}{5}\cdot 10^2 = 18(2)(10)$

$= 360\ \frac{ft}{s}$?!

Why's that look wrong?

$3^2(1000)^2 + 4^2(1000)^2$

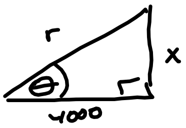
$= (3^2+4^2)(10^6)$

$= 25(10)^6$

I *think* *dr/dt* should be GREATER than *dx/dt*.

Book agrees w/ (a) $\boxed{\left.\frac{dr}{dt}\right|_{x=3000} = 360\ \frac{ft}{s}}$, but I have

(b) We find $\left.\frac{d\theta}{dt}\right|_{x=3000}$:



$\Longrightarrow \frac{x}{4000} = \tan\theta$

$x = 4000 \tan\theta$

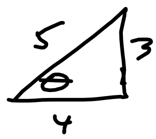
$\frac{dx}{dt} = 4000 \sec^2\theta \, \frac{d\theta}{dt}$, by chain rule on $\theta = \theta(t)$.

we need θ:

$\theta = \arctan\left(\frac{3000}{4000}\right) = \arctan\left(\frac{3}{4}\right)^{*} \approx$ 0.6435011088 $\longrightarrow$

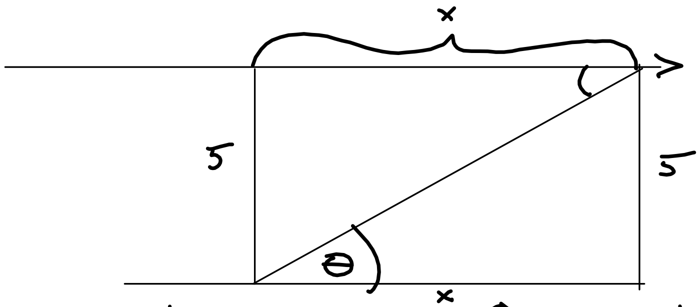
$* \; 0 < \theta < \frac{\pi}{2}$

$600 \approx 4000 \sec^2(.6435011088)^2 \left.\frac{d\theta}{dt}\right|_{x=3000}$



$\Longrightarrow \left.\frac{d\theta}{dt}\right|_{x=3000} \approx \frac{600}{4000 \sec^2(.64\ldots)} \approx$ $\boxed{0.09600000000 \; \frac{\text{radians}}{\text{sec}} \approx \left.\frac{d\theta}{dt}\right|_{x=3000}}$

45. A plane flies horizontally at an altitude of 5 km and passes directly over a tracking telescope on the ground. When the angle of elevation is $\pi/3$, this angle is decreasing at a rate of $\pi/6$ radians per minute. How fast is the plane traveling at that time? → want $\frac{dx}{dt}$ when $\theta = \frac{\pi}{3}$



Let x = distance (horizontal) from the tower (in km),
θ = angle of elevation to the plane (in radians), &
t = time (minutes).

Given $\left.\frac{d\theta}{dt}\right|_{\theta=\frac{\pi}{3}} = -\frac{\pi}{6}$ radians/minute

From figure: $\frac{5}{x} = \tan\theta$

$$x = \frac{5}{\tan\theta} = 5\cot\theta$$

$$\frac{dx}{dt} = 5(-\csc^2\theta)\left(\frac{d\theta}{dt}\right)$$

$$\left.\frac{dx}{dt}\right|_{\theta=\frac{\pi}{3}} = -5\left(\csc^2\left(\frac{\pi}{3}\right)\right)\left(-\frac{\pi}{6}\right)$$

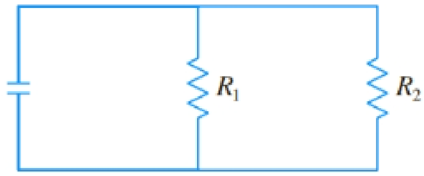
$$= \frac{5\pi}{6}\left(\frac{2}{\sqrt{3}}\right)^2$$

$$= \frac{5\pi\cdot 2^2}{6\cdot 3} = \frac{10\pi}{3\cdot 3} = \boxed{\frac{10\pi}{9}\ \frac{\text{km}}{\text{min}}}$$

39. If two resistors with resistances R_1 and R_2 are connected in parallel, as in the figure, then the total resistance R, measured in ohms (Ω), is given by

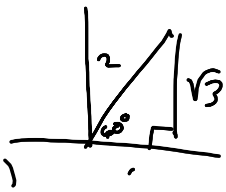
$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

If R_1 and R_2 are increasing at rates of $0.3\ \Omega/\text{s}$ and $0.2\ \Omega/\text{s}$, respectively, how fast is R changing when $R_1 = 80\ \Omega$ and $R_2 = 100\ \Omega$?



2.9 Stuff

Find the tangent line to $\tan(x)$ **at** $x = \frac{\pi}{3} = 60^\circ$



$f(x) = \tan(x)$ $\quad f(\frac{\pi}{3}) = \tan(\frac{\pi}{3}) = \sqrt{3}$

$f'(x) = \sec^2(x)$ $\quad f'(\pi/3) = \sec^2(\frac{\pi}{3}) = 2^2 = 4 = m = f'(x_1)$

$x_1 = 60^\circ = \frac{\pi}{3}$

$x_2 = 57^\circ$

$L_{\frac{\pi}{3}}(x) = 4(x - \frac{\pi}{3}) + \sqrt{3}$

Use your result to estimate $\tan(57^\circ)$

$x_2 = (57^\circ)(\frac{\pi}{180^\circ}) =$

$= (60^\circ - 3^\circ)(\frac{\pi}{180^\circ}) = \frac{\pi}{3} - \frac{\pi}{60}$

$L_{\frac{\pi}{3}}(57^\circ(\frac{\pi}{180^\circ})) = 4(\frac{\pi}{3} - \frac{\pi}{60} - \frac{\pi}{3}) + \sqrt{3}$

$= 4(\frac{-\pi}{60}) + \sqrt{3}$

$= -\frac{\pi}{15} + \sqrt{3}$

$-\frac{\pi}{60} = \Delta x = dx$

$L(x) = \boxed{f'(x_1)(x - x_1)} + f(x_1)$

$= \boxed{f'(x_1)\Delta x} + f(x_1)$

$= \boxed{f'(x_1)dx} + f(x_1)$

$f(x_2) \approx f'(x_1)dx + f(x_1)$

$f(x_2) - f(x_1) = \Delta y \approx dy = \boxed{f'(x_1)dx}$