Epsilon-Delta Revisited.

Section 2.9 ideas.

Questions?

Prove $\lim_{x\to 5}(-3x+2) = -13$

Need Abs. Value, here, Steve!

Proof:

Let $\varepsilon > 0$. Define $\delta = \frac{\varepsilon}{3}$. Then $0 < |x-5| < \delta \implies$

$$|-3x+2-(-13)| = |-3x+15| = |-3||x-5| = 3|x-5| < 3\delta = 3\cdot\frac{\varepsilon}{3} = \varepsilon \blacksquare$$

Bonus: Claim: $\lim_{x\to 5}(x^2-3x+2) = 12$

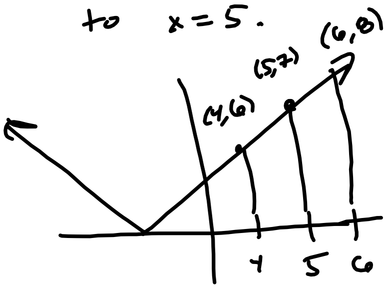
Scratch:

$$|x^2-3x+2-12| = |x^2-3x-10| = \underbrace{|x-5|}_{<\delta}\underbrace{|x+2|}_{<?}$$

JQ

Need an upper bound for the $|x+2|$.

Assume we're "close" to $x=5$.



$|x-5| < \delta$

If $\delta \le 1$, then

Should all be "<"!

$4 \le x \le 6$

$6 \le x+2 \le 8$

$\implies |x+2| \le 8$ Yes!

$<$

&

$|x+2| \le 8$

Proof:

Let $\varepsilon > 0$ be given. Define $\delta = \min\{1, \frac{\varepsilon}{8}\}$. Then if $0 < |x-5| < \delta$, we have $|x^2 - 3x + 2 - 12|$

$= |x^2 - 3x - 10| = |x+2||x-5| \le 8|x-5| < 8\delta \le 8 \cdot \frac{\varepsilon}{8} = \varepsilon$ ∎

$<$

$0 < |x-5| < \delta \Rightarrow$

$-1 \le -\delta < x-5 < \delta \le 1$

$-1+7 < x-5+7 < 1+7$

$6 < x+2 < 8$

i.e., $|x+2| < 8$ & that's what we need for $\delta = \min\{1, \frac{\varepsilon}{8}\}$

Questions on anything, including the Writing Project?

Linearization is just a fancy name for the tangent line.

Given $f(x)$, $f'(x)$,

$$L_a(x) = f'(a)(x-a) + f(a)$$

Find $L_1(x)$ for $f(x) = \sqrt{x} = x^{\frac{1}{2}}$.

$$f(1) = \sqrt{1} = 1 \rightarrow \boxed{(1,1) = (a, f(a))}$$

$$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} \rightarrow$$

$$f'(a) = f'(1) = \frac{1}{2}(1)^{-\frac{1}{2}} = \boxed{\frac{1}{2} = f'(a)}$$

$$L_a(x) = L_1(x) = f'(a)(x-a) + f(a)$$

$$= \frac{1}{2}(x-1) + 1$$

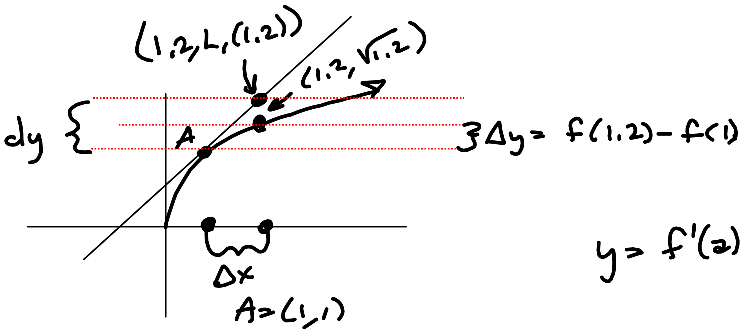
Use the above to approximate $\sqrt{1.2}$

$$L_1(1.2) = \frac{1}{2}(1.2-1) + 1 = \frac{1}{2}(.2) + 1 = \frac{1}{2}\left(\frac{2}{10}\right) + 1$$

$$= \frac{1}{10} + 1 = \frac{11}{10} = \boxed{1.1 \approx \sqrt{1.2}}$$

Not bad.

Actual $\boxed{\sqrt{1.2} \approx 1.095445115}$



$$y = f'(a)\underbrace{(x-a)}_{\Delta x = dx} + f(a)$$

Define!

$$y = f'(a)(x-a) + f(a) \approx f(x)$$

$$f(x) \approx f'(a)(x-a) + f(a)$$

$$f(x) - f(a) \approx f'(a)(x-a)$$

$$\Delta y = f(x) - f(a) = f'(a)\Delta x = f'(a)dx$$

$$= dy$$

You want to estimate how much paint it takes to make a .1 cm coat on a dome (hemispherical) with radius $r = 10m$.

Here's the idea:

$V = \frac{4}{3}\pi r^3$, we want half:

$= \frac{2}{3}\pi r^3$

Increase the radius by 0.1 cm

?

What's the change in volume

$10\text{m} + .1\text{cm} = 10\text{m} + .001\ \text{m} = 10.001\ \text{m}$, so the volume of the paint is

$$V(10.001) - V(10)$$

$$\frac{2\pi}{3}(10.001)^3 - \frac{2\pi}{3}(10)^3$$

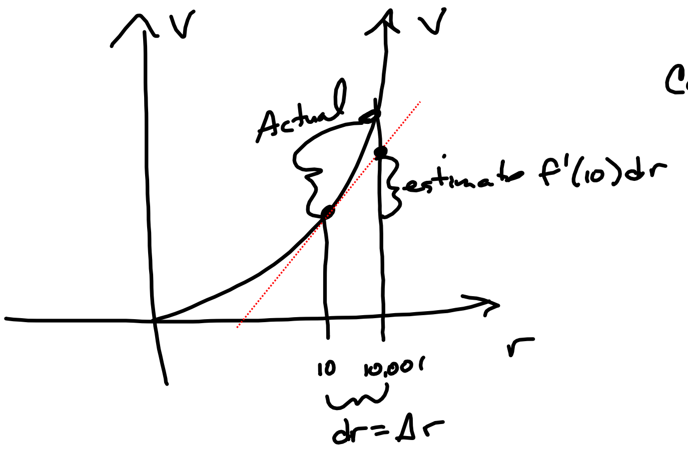
$$= \frac{2\pi}{3}\left[(10.001)^3 - 10^3\right] \approx 0.6283813627\ m^3$$ BIGGER

$$V = \frac{2}{3}\pi r^3$$

$$\frac{dV}{dr} = 2\pi r^2$$

$$\Delta V \approx dV = 2\pi r^2 dr$$

$$= 2\pi(10)^2(.001) \approx 0.6283185308$$ SMALLER



Concave up ☺

Tangent below graph

This stuff is putting us right on the edge of Taylor's and Maclaurin's series.

Tangent line to sine @ $x=c$:

$f'(x) = \cos(x) \longrightarrow$ $\sin(c)$

$L_c(x) = \cos(c)(x-c) + \sin(c)$

Tangent line is 1st degree Taylor Approximation/Polynomial.

$\sin(33^\circ)$

$33 = 30 + 3$

$3 = \Delta x$

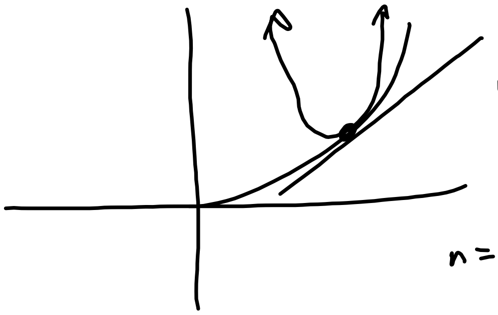
$c = 30 \cdot \frac{\pi}{180^\circ} = \frac{\pi}{6}$

$$\sin(33^\circ) = \sin\left(\frac{\pi}{6} + \frac{\pi}{60}\right)$$

$$\approx \sin(30^\circ) + \cos(30^\circ)\left(\frac{\pi}{6} + \frac{\pi}{60} - \frac{\pi}{6}\right)$$

$$= \frac{1}{2} + \frac{\sqrt{3}}{2}\left(\frac{\pi}{60}\right) \approx \sin(33^\circ)$$

Linearization $n=1$



$n=2$ Fit quadratic

$n=3$: 3rd degree Taylor polynomial