Writing Project/Written Test Stuff

#s 1 - 3: Compute the limits

1 $\lim\limits_{x\to -7} \dfrac{x^2+10x+21}{x^2-5x-84} = \lim\limits_{x\to -7} \dfrac{\cancel{(x+7)}(x+3)}{\cancel{(x+7)}(x-12)} = \lim\limits_{x\to -7} \dfrac{x+3}{x-12} = \dfrac{-4}{-19}$

$\boxed{= \dfrac{4}{19}}$

$$\begin{array}{r|l} 2 & 84 \\ 2 & 42 \\ 3 & 21 \\ & 7 \end{array}$$

2a $\lim\limits_{x\to -3^-} \dfrac{x^2-8x-33}{|x^2+3x|}$

$= \lim\limits_{x\to -3^-} \dfrac{(x+3)(x-11)}{-x(-(x+3))} = \lim\limits_{x\to -3^-} \dfrac{x-11}{x}$

$= \dfrac{-3-11}{-3} = \dfrac{-14}{-3} = \boxed{\dfrac{14}{3}}$

$|x^2+3x| = |x||x+3|$

$\xrightarrow{x\to -3^-} -x(-(x+3))$

because

$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

$\& -3 < 0$

$|x+3| = \begin{cases} x+3 & \text{if } x \geq -3 \quad \cancel{x+3 \geq 0} \\ -(x+3) & \text{if } x < -3 \end{cases}$

(from $x+3 < 0$)

2b $\displaystyle \lim_{x\to -3^+} \frac{x^2-8x-33}{|x^2+3x|}$

$|x^2+3x| = |x||x+3|$

$x \to -3^+ \longrightarrow +x\,(+(x+3))$

because
$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

& $-3 < 0$

$|x+3| = \begin{cases} x+3 & \text{if } x \geq -3 \quad \cancel{x+3 \geq 0} \\ -(x+3) & \text{if } x < -3 \end{cases}$

(from $x+3 < 0$)

$\displaystyle = \lim_{x\to -3^+} \frac{(x+3)(x-11)}{x(x+3)}$

$\displaystyle = \lim_{x\to -3^+} \frac{x-11}{x} = \frac{-3-11}{-3} \boxed{= \frac{14}{3}}$

2c $\displaystyle \lim_{x\to -3} \frac{x^2-8x-33}{|x^2+3x|} = \frac{14}{3}$, b/c $\displaystyle \lim_{x\to -3^-} \text{[illegible]} = \lim_{x\to -3^+} \text{[illegible]} = \frac{14}{3}$

$$\lim_{x\to -3} \frac{x^2-8x-33}{|x+3|} = \lim f$$

$$\lim_{x\to -3^-} f = \lim_{x\to -3^-} \frac{(x+3)(x-11)}{-(x+3)} = \lim_{x\to -3^-} \frac{x-11}{-1} = \boxed{+14}$$

$$\lim_{x\to -3^+} f = \lim_{x\to -3^+} \frac{(x+3)(x-11)}{x+3} = \lim_{x\to -3^+} (x-11) = \boxed{-14}$$

$$\Rightarrow \lim f \nexists \text{ b/c } \lim_{x\to -3^-} f = 14 \neq -14 = \lim_{x\to -3^+} f.$$

3 $\lim_{h \to 0}\left(\frac{1}{h}\left(\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}\right)\right)$

$$\frac{1}{h}\left[\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}\right] = \left(\frac{\sqrt{x}-\sqrt{x+h}}{\sqrt{x}\sqrt{x+h}}\right)\left(\frac{\sqrt{x}+\sqrt{x+h}}{\sqrt{x}+\sqrt{x+h}}\right) = \frac{x-(x+h)}{\cdots} = \frac{-h}{\cdots}$$

$$\xrightarrow{h \to 0} \frac{-1}{\sqrt{x}\sqrt{x+h}\,(\sqrt{x}+\sqrt{x+h})}$$

4. Let $f(x) = \frac{1}{\sqrt{x}}$. Find $f'(x)$ using the limit definition.

$\boxed{\text{See \#3}}$

5 Let $f(x) = \frac{1}{\sqrt{x}}$. Find $f'(x)$ using the power rule.

$$f(x) = \frac{1}{x^{\frac{1}{2}}} = x^{-\frac{1}{2}} \longrightarrow \boxed{f'(x) = -\frac{1}{2}x^{-\frac{3}{2}}}$$

6. Let $f(x) = \frac{1}{\sqrt{\sin(x)}}$. Find $f'(x)$ using the General Power Rule.

$nf^{n-1}\frac{df}{dx}$

$$\Rightarrow f(x) = (\sin(x))^{-\frac{1}{2}}$$

$$\Rightarrow f'(x) = -\frac{1}{2}(\sin(x))^{-\frac{3}{2}}(\cos(x))$$

7. Let $f(x) = \frac{1}{\sqrt{\sin(x)}}$. Find $f'(x)$ using the Chain Rule

See #6: We just did, because the General Power Rule is just a special case of the Chain Rule.

#s 8 - 12: Compute the indicated derivative

(8) $f(x) = \overset{f}{(x\sin(x))}\overset{g}{(x^2-1)^2} \Longrightarrow f'(x) =$

$\underbrace{(1\sin(x) + x\cos(x))}_{f'}\underbrace{(x^2-1)^2}_{g} + \underbrace{(x\sin(x))}_{f}\underbrace{(2(x^2-1)'(2x))}_{g'}$

g' required Chain Rule.

(9) $f(x) = (x\sin(x))(x^2-1)^2 \Longrightarrow \frac{df}{dx} = \boxed{\text{See \#8}}$

(10) $f(x) = (x\sin(x))(x^2-1)^2 \Longrightarrow \frac{df}{dg} =$

Kind of 2 ways to interpret this: Is *x* implicitly a function of *g*?

$\left[\underbrace{(1\sin(x) + x\cos(x))}_{f'}(x^2-1)^2 + (x\sin(x))(2(x^2-1)'(2x))\right]\frac{dx}{dg}$

$V = \frac{1}{3}\pi r^2 h$ = Vol of cone. $(fg)' = f'g + fg'$

$\Longrightarrow \frac{dV}{dt} = \frac{2}{3}\pi r \frac{dr}{dt} h + \frac{1}{3}\pi r^2 \frac{dh}{dt}$ $f = r^2, g = h$

Is *x* an independent variable, and *g* is just another variable? $\frac{df}{dg} = 0$

$= \frac{1}{3}\pi\left[r\frac{dr}{dt}h + r^2\frac{dh}{dt}\right]$

$= \frac{1}{3}\pi\left[rr'h + r^2h'\right]$

11. $\frac{d}{dx}\left[(x\sin(x))(x^2-1)^2\right] =$ **See above. I'm tired.**

12. $\frac{d}{d\theta}\left[(x\sin(x))(x^2-1)^2\right] = 0$, if x is independent b/c $\frac{dx}{d\theta} = 0$

$$\left[\overbrace{(1\sin(x) + x\cos(x))(x^2-1)^2 + (x\sin(x))(2(x^2-1)^1(2x))}^{\frac{df}{dx}}\right]\overbrace{\frac{dx}{d\theta}}^{\frac{dx}{d\theta}}$$

if x depends on θ in some way.