Today:

2.5 - Chain Rule ✓

2.6 - Implicit Differentiation ✓

2.7 - Applications to Science

2.8 - Related Rates

Picking up the spare:

Recall: we needed $\lim_{h \to 0} \frac{\cos(h)-1}{h} = 0$

It's the missing piece of $\frac{d}{dx}[\sin(x)] = \cos(x)$.

$$\frac{\cos(h)-1}{h} = \frac{(\cos(h)-1)(\cos(h)+1)}{h(\cos(h)+1)} = \frac{\cos^2(h)-1}{h(\cos(h)+1)} = \frac{-\sin^2(h)}{h(\cos(h)+1)}$$

$$= \frac{-\sin(h)}{h} \cdot \frac{\sin(h)}{\cos(h)+1} \xrightarrow{h \to 0} 1 \cdot \frac{0}{2} = 0.$$

$$\frac{\sin(x+h)-\sin(x)}{h} = \frac{\sin(x)\cos(h)+\sin(h)\cos(x)-\sin(x)}{h}$$

$$= \frac{\sin(x)(\cos(h)-1)}{h} + \frac{\sin(h)}{h}\cos(x)$$

$$= \sin(x)\left(\frac{\cos(h)-1}{h}\right) + \frac{\sin(h)}{h}\cos(x) \xrightarrow{h \to 0} \cos(x) \blacksquare$$

§ 2.5

$$\frac{d}{dx}\left[\text{outside}(\text{inside}(x))\right] = \frac{d}{dx}\left[f(g(x))\right]$$

$$= \frac{d\,\text{outside}}{d\,\text{inside}} \cdot \frac{d\,\text{inside}}{dx} = \frac{df}{dg} \cdot \frac{dg}{dx}$$

$$\frac{d}{dx}\left[\sin(\cos(x))\right] = \cos(\cos(x)) \cdot (-\sin(x))$$

$$\frac{d}{dx}\left[\sin(x^2-3x)\right] = \underbrace{\cos(x^2-3x)}_{\frac{d\sin(x^2-3x)}{d(x^2-3x)}} \cdot \underbrace{(2x-3)}_{\frac{d(x^2-3x)}{dx}} = \frac{df}{dg} \cdot \frac{dg}{dx}$$

where $f = \sin(g)$ $\quad \frac{df}{dg} = \cos(g)$

$g = x^2 - 3x$

$$\frac{d}{dx}\left[\left(\sin(x) + x^2\right)^{17}\right]$$

$$= 17\left(\sin(x) + x^2\right)^{16}(\cos(x) + 2x)$$

$$\frac{d}{dx}\left[\sin(3x)\right] = \underbrace{\cos(3x)\cdot 3}_{\text{By rule}} = \underbrace{3\cos(3x)}_{\text{style}}$$

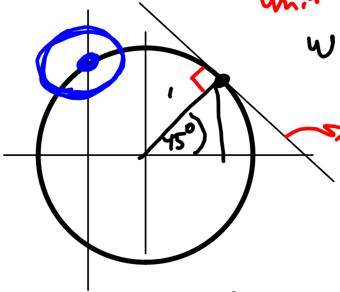
§2.6

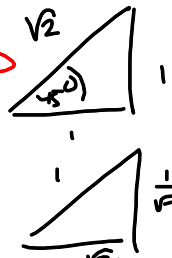
Suppose y is a function of x.

$$\frac{d}{dx}\left[y^2\right] = 2y \cdot \frac{dy}{dx} = 2yy'$$

Unit Circle

what's the slope @ $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$?





$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\rightarrow m = -1$ by "skills."

"Locally," this IS a function, $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

so treating y implicitly as a function of x works "locally."

OLD WAY:

$x^2 + y^2 = 1$

$y^2 = 1 - x^2$

$y = \pm\sqrt{1-x^2}$

$\nearrow$ $y = \sqrt{1-x^2}$ top $\frac{1}{2}$

$\searrow$ $y = -\sqrt{1-x^2}$ bottom $\frac{1}{2}$

Want $\frac{dy}{dx}$ @ $(x,y) = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$$\frac{dy}{dx} = \frac{d}{dx}\left[(1-x^2)^{\frac{1}{2}}\right] = \frac{1}{2}(1-x^2)^{-\frac{1}{2}}(-2x) = \frac{-x}{\sqrt{1-x^2}}$$

$$\left.\frac{dy}{dx}\right|_{\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)} = \frac{-\frac{1}{\sqrt{2}}}{\sqrt{1-\left(\frac{1}{\sqrt{2}}\right)^2}} = \frac{-\frac{1}{\sqrt{2}}}{\sqrt{\frac{2-1}{2}}} = \frac{-\frac{1}{\sqrt{2}}}{\sqrt{\frac{1}{2}}} = \frac{-\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = -1$$

$m = -1$, as expected.

2.6 Way

$$x^2 + y^2 = 1$$

$$\frac{d}{dx}\left[x^2+y^2\right] = \frac{d}{dx}\left[1\right]$$

$$2x + 2y \cdot y' = 0$$

$$2yy' = -2x$$

$$yy' = -x$$

$y' = -\frac{x}{y}$ @ $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$, $y' = -\frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = -1 = m$!

Textbook #s 25–32

#s 25 - 32: Find an equation of the tangent line to the curve at the given point.

$\sin(x+y) = 2x - 2y$ @ (π, π)

$\Rightarrow \cos(x+y)(1+y') = 2 - 2y'$

$\Rightarrow \cos(x+y)\,y' + \cos(x+y) = 2 - 2y'$

$\Rightarrow \cos(x+y)\,y' + 2y' = 2 - \cos(x+y)$

$\Rightarrow y'(\cos(x+y) + 2) = 2 - \cos(x+y)$

$\Rightarrow y' = \dfrac{2 - \cos(x+y)}{\cos(x+y) + 2}$

$\Rightarrow y'\Big|_{\substack{x=\pi \\ y=\pi}} = \dfrac{2 - \cos(\pi+\pi)}{\cos(\pi+\pi) + 2} = \dfrac{2-1}{1+2} = \dfrac{1}{3} = m$

$L(x) = \boxed{y = \frac{1}{3}(x-\pi) + \pi}$

$= y'\Big|_{(\pi,\pi)} (x - \pi) + y\Big|_{\substack{x=\pi \\ y=\pi}}$

$= m(x - x_1) + y_1$ where

$m = y'\Big|_{(\pi,\pi) = (x,y)}$, $x_1 = \pi$, $y_1 = \pi$

WebAssign S2.6 #9. Same instructions

$$x^{2/3}+y^{2/3}=4 \qquad (x_1,y_1)=(-3\sqrt{3},1)$$

$$\rightarrow \frac{2}{3}x^{-\frac{1}{3}}+\frac{2}{3}y^{-\frac{1}{3}}\cdot y'=0$$

$$\rightarrow \frac{2}{3}y^{-\frac{1}{3}}y'=-\frac{2}{3}x^{-\frac{1}{3}}$$

$$y^{-\frac{1}{3}}y'=-x^{-\frac{1}{3}}=\frac{1}{-x^{1/3}} \qquad \frac{3}{2}\cdot\frac{1}{3}=\frac{1}{2}$$

$$y'=\frac{y^{\frac{1}{3}}}{-x^{\frac{1}{3}}}=\frac{1^{\frac{1}{3}}}{-(3\sqrt{3})^{\frac{1}{3}}}=\frac{1}{-(3^{\frac{3}{2}})^{1/3}}=\frac{1}{-3^{\frac{1}{2}}}=\boxed{-\frac{1}{\sqrt{3}}=m}$$

$-3\sqrt{3}$ $+$ $+$ $+$

$$y=\frac{1}{\sqrt{3}}(x+3\sqrt{3})+1$$

2.6 #13 **Find where the curve has a horizontal tangent.**

$$2(x^2+y^2)^2=81(x^2-y^2)$$
$$=81x^2-81y^2$$

$$4(x^2+y^2)(2x+2yy')=162x-162yy'$$

$$4(x^2+y^2)(2x)+4(x^2+y^2)(2yy')=162x-162yy'$$

$$4x(x^2+y^2)+4yy'(x^2+y^2)=81x-81yy'$$

$$4yy'(x^2+y^2)+81yy'=81x-4x(x^2+y^2)$$

$$y' = \frac{81x - 4x(x^2+y^2)}{4y(x^2+y^2)+81y} \quad \text{SET} = 0$$

$\Rightarrow 81x - 4x(x^2+y^2) = 0$

$81x - 4x^3 - 4xy^2 = 0$ is not trivial.

$x(81 - 4x^2 - 4y^2) = 0$

$x = 0$ OR $81 - 4x^2 - 4y^2 = 0$

$4x^2 + 4y^2 = 81$

$x^2 + y^2 = \frac{81}{4}$ is eq'n of a circle of radius $\frac{9}{2}$ centered @ $(0,0)$

So solutions are on that circle's intersection with the lemniscate (curve)

Aha! Now we can substitute 81/4 for x^2 + y^2 in the equation for the lemniscate!

$2(x^2+y^2)^2 = 81(x^2-y^2) \Rightarrow$

$2\left(\frac{81}{4}\right)^2 = 81(x^2-y^2) \Rightarrow$

$\frac{81^2}{8} = 81(x^2-y^2) \Rightarrow$

$\frac{81}{8} = x^2 - y^2 \longrightarrow$

$x^2+y^2 = \frac{81}{4}$

$x^2 = \frac{81}{4} - y^2$

$y^2 = x^2 - \frac{81}{8}$

$\cancel{y = \pm\sqrt{x^2 - \frac{81}{8}}}$ $= \pm\sqrt{}$

$y^2 = \frac{81}{4} - y^2 - \frac{81}{8}$

$2y^2 = \frac{81}{8}$ using $x^2+y^2 = \frac{81}{4}$, again.

$y^2 = \frac{81}{16}$

$\boxed{y = \pm\frac{9}{4}}$ so $x = \ldots$

$3 \,\overline{)243}$

$9 \,\overline{)81}$

9

$x^2+y^2 = \frac{81}{4}$

$x^2 + \frac{81}{16} = \frac{81}{4}$

$x^2 = \frac{81}{4}\cdot\frac{4}{4} - \frac{81}{16} = \frac{3(81)}{16} = \frac{243}{16}$

$\Rightarrow x = \pm\sqrt{\frac{243}{16}} = \boxed{\pm\, 9\sqrt{3} = x}$

$x^2 - 3x + 2$

$a = 1, b = -3, c = 2$

$b^2 - 4ac = (-3)^2 - 4(1)(2) = 9 - 8$

$= 3^2 - 4(1)(2) = 9 - 8$

$-3^2 - 4(1)(2)$ → No

$= 9 - 8$ → No