$$\lim_{x\to 7} \frac{x^2+4x-77}{x^3-4x^2-19x-14} = \frac{0}{0}$$

$7^2+4(7)-77=0$

$7^3-4(7)^2-19(7)-14 = 343-196-133-14$

$= 343-343=0$

$$\begin{array}{r} {}^{6}49 \\ 7 \\ \hline 343 \end{array} \quad \begin{array}{r} {}^{3}49 \\ 4 \\ \hline 196 \end{array} \quad \begin{array}{r} {}^{6}19 \\ 7 \\ \hline 133 \end{array}$$

$-329-14$

$=-343$

Evaluating polynomials by synthetic division:

For $f(7)$, divide $f(x)$ by $x-7$. The remainder is $f(7)$.

$$\frac{f(x)}{x-7} = q(x) + \frac{r(x)}{x-7} \implies f(x) = (x-7)q(x)+r(x)$$

$$\implies f(7) = r(7)!$$

$$\frac{27}{8} = 3 + \frac{3}{8}$$

$$\begin{array}{r} 3\ r3 \\ 8\overline{)27} \\ -24 \\ \hline 3 \end{array}$$

$\implies 27 = 3(8)+3$

$$
\begin{array}{r}
x^2+3x+2 \quad r\,0 \\
x-7 \overline{\big)\, x^3-4x^2-19x-14} \\
-\,(x^3-7x^2 \qquad\qquad) \\
\hline
3x^2-19x-14 \\
-\,(3x^2-21x \qquad) \\
\hline
2x-14 \\
2x-14 \\
\hline
0
\end{array}
$$

This says

$$x^3-4x^2-19x-14 = (x-7)(x^2+3x+2)+0$$

i.e., $x-7$ is a factor.

Quick way!

$$
\begin{array}{r|rrrr}
7 & 1 & -4 & -19 & -14 \\
 & & 7 & 21 & 14 \\
\hline
 & 1 & 3 & 2 & 0 \rightarrow \text{Sweet!} \\
 & x^2 & x & c & r
\end{array}
$$

$\Rightarrow$

$f(x) = x^3-4x^2-19x-14 = (x-7)(x^2+3x+2)$

& $f(7)=0$!

$f(3)$ for $x^3-4x^2-19x-14$

$$
\begin{array}{r|rrrr}
3 & 1 & -4 & -19 & -14 \\
 & & 3 & -3 & -66 \\
\hline
 & 1 & -1 & -22 & -80 \\
 & x^2 & x^1 & c & r
\end{array}
$$

$f(3) = -80$

$x^3-4x^2-19x-14$

$= (x-3)(x^2-x-22)-80$

$\Rightarrow f(3) = -80$

$$\lim_{x\to 7} \frac{x^2+4x-77}{x^3-4x^2\,19x-14} = \frac{0}{0} = \lim_{x\to 7} \frac{(x-7)(x+11)}{(x-7)(x^2+3x+2)} = \lim_{x\to 7} \frac{x+11}{x^2+3x+7}$$

$$= \frac{7+11}{49+21+7} = \boxed{\frac{18}{77}}$$

$7^2+4(7)-77=0$

$7^3-4(7)^2-19(7)-14 = 343-196-133-14$

$= 343-343=0$

$$\begin{array}{r|rrr} 7 & 1 & 4 & -77 \\ & & 7 & 77 \\ \hline & 1 & 11 & 0 \end{array}$$

$(x-7)(x+11)$

$$\begin{array}{r|rrrr} 7 & 1 & -4 & -19 & -14 \\ & & 7 & 21 & 14 \\ \hline & 1 & 3 & 2 & 0 \end{array}$$

$(x-7)(x^2+3x+2)$

11. For what values of x does the graph of f have a horizontal tangent? (Use n as your integer variable. Enter your answers as a comma-separated list.)

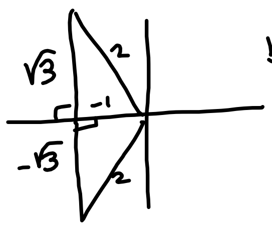
$f(x) = x + 2\sin(x)$

$f'(x) = 1 + 2\cos(x) \overset{SET}{=} 0 \Longrightarrow$

$\cos(x) = -\frac{1}{2}$

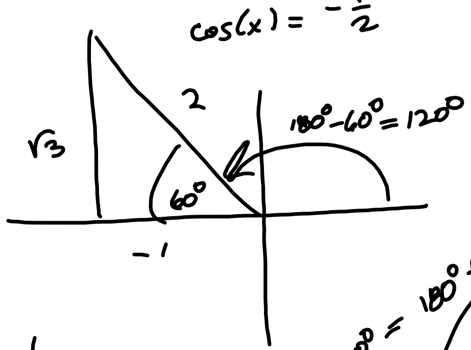
$2\cos(x) = -1$

$\cos(x) = -\frac{1}{2}$

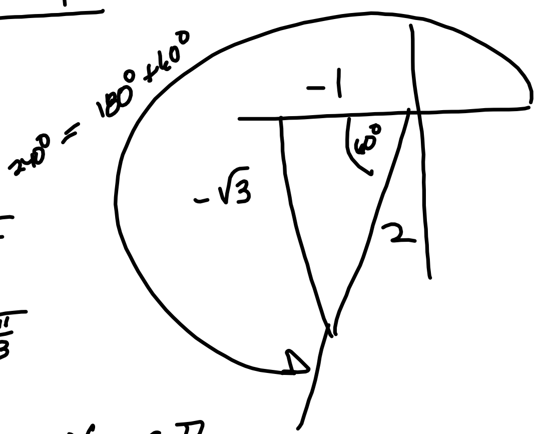


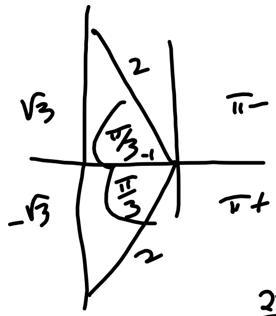
$b^2 = 2^2 - 1^2 = 3 \Longrightarrow$

$b = \sqrt{3}$



60° reference angle.





$\pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$\pi + \frac{\pi}{3} = \frac{4\pi}{3}$

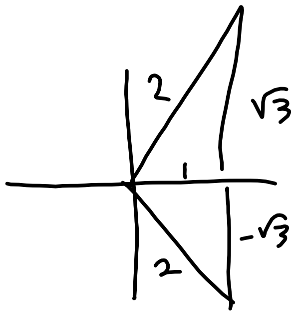
$\frac{2\pi}{3} + 2n\pi \quad \forall\, n \in \mathbb{Z}$

$\frac{4\pi}{3} + 2n\pi \quad$ " " " "

2.4 #11

$f(x) = x - 2\sin x$ version:

$\Rightarrow f'(x) = 1 - 2\cos(x) \overset{SET}{=} 0$



$2\cos(x) = 1$

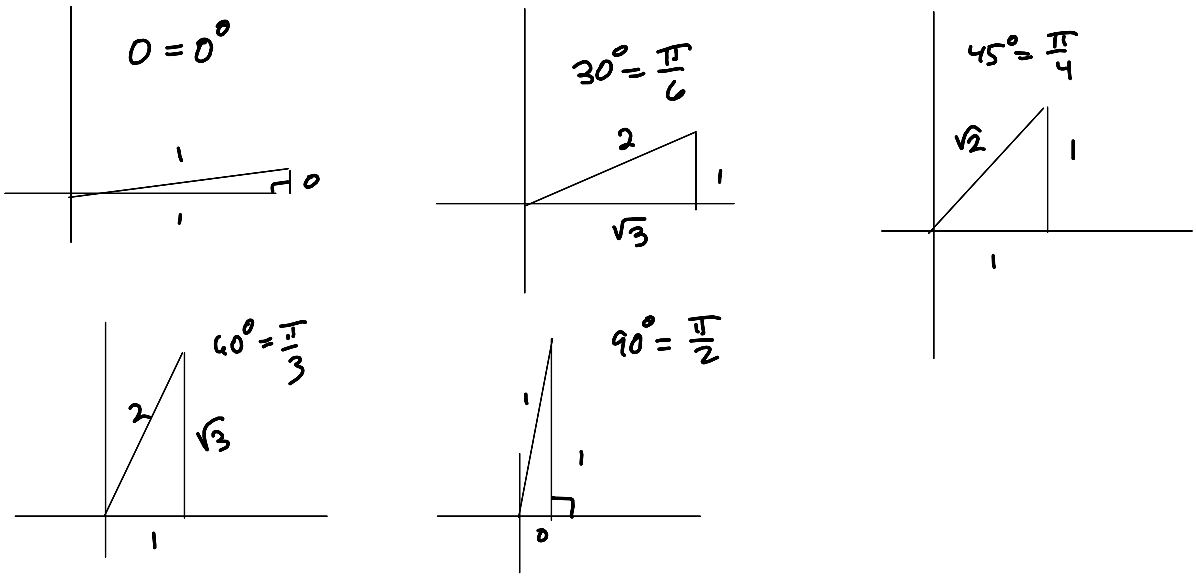
$\cos(x) = \frac{1}{2}$

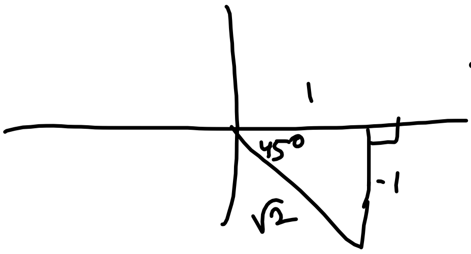
$x = \frac{\pi}{3}, \frac{5\pi}{3}$ solves it

on $[0, 2\pi]$.

solns, use period $= 2\pi$

$x = \frac{\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi$





$$\sin(315^\circ) = \sin\left(\frac{7\pi}{4}\right) = -\frac{1}{\sqrt{2}}$$

$f(x) = \dfrac{\cos(x)}{\sin(x)+2}$ 2.4 #12 Horizontal Tangent(s)

$f'(x) = \dfrac{-\sin(x)(\sin(x)+2) - \cos(x)\cos(x)}{(\sin(x)+2)^2}$ SET $= 0$

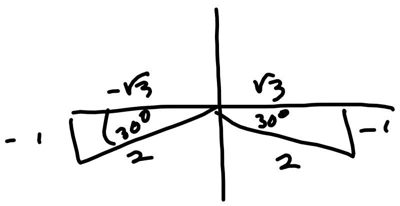
$\Rightarrow -\sin^2(x) - 2\sin(x) - \cos^2(x) = 0$

$\Rightarrow -(\sin^2(x) + \cos^2(x)) - 2\sin(x)$

$= -1 - 2\sin(x) = 0$

$\Rightarrow 2\sin(x) = -1$

$\sin(x) = -\frac{1}{2}$

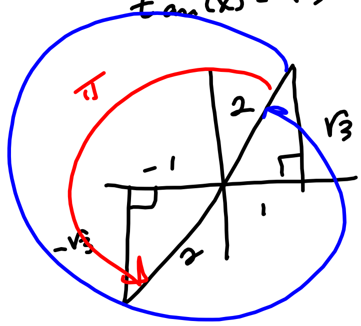


$180^\circ + 30^\circ = 210^\circ$ $\pi + \frac{\pi}{6} = \frac{7\pi}{6}$

$360^\circ - 30^\circ = 330^\circ$ $2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$

$x = \frac{7\pi}{6} + 2n\pi, \ \frac{11\pi}{6} + 2n\pi$

$\tan(x) = \sqrt{3}$



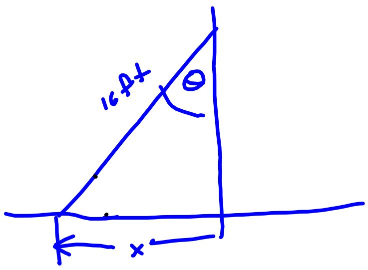
$\frac{\pi}{3}, \ \pi + \frac{\pi}{3} = \frac{4\pi}{3}$

$x = \frac{\pi}{3} + n\pi$ does it all! (Tangent has period π)

so does:

$\frac{\pi}{3} + 2n\pi, \ \frac{4\pi}{3} + 2n\pi$

2.4 #14

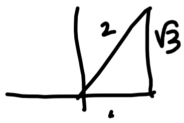


What's $\left.\dfrac{dx}{d\theta}\right|_{\theta=\frac{\pi}{3}}$?

What's the rate of change of *x* with respect to (an incremental change in) θ?

$\dfrac{x}{16} = \sin\theta$

$x = 16\sin\theta$



$$\left.\frac{dx}{d\theta}\right|_{\theta=\frac{\pi}{3}} = 16\cos\theta\Big|_{\theta=\frac{\pi}{3}} = 16\cos\left(\frac{\pi}{3}\right) = 16\left(\frac{1}{2}\right) = 8\ \frac{\text{ft}}{\text{radian}}$$

Do Not factor out the coefficient of *x* inside the sine for these...

$$\lim_{x\to 0} \frac{\sin(x)}{x} = 1$$

?!

$$\lim_{x\to 0} \frac{\sin(3x)}{\sin(5x)} = \boxed{\lim_{x\to 0} \frac{3\sin(x)}{5\sin(x)}} = \lim_{x\to 0} \frac{3}{5} = \frac{3}{5}$$

No!

$$\lim_{x\to 0}\left(\frac{3x\sin(3x)}{3x} \cdot \frac{5x}{5x\sin(5x)}\right) = \lim_{x\to 0} \frac{3x}{5x}\left(\frac{\sin(3x)}{3x}\right)\left(\frac{5x}{\sin(5x)}\right)$$

$$= \frac{3}{5}\cdot 1 \cdot 1 = \frac{3}{5}$$

The profit on x purses sold is 25x. $= P(x)$

The production of x purses is $x =$ $30t$

where $t =$ time, in hours.
$x = \#$ of purses sold.

What's the rate of profit with respect to time?

$$\frac{d}{dt}\left[25x\right] = \frac{d}{dt}\left[25(30t)\right] = 750\ \frac{\$}{hr}$$

Chain Rule:

$x = x(t) = g(t)$

Profit as a function of time is

$25(x(t)) = 25g(t) \longrightarrow$

$$\frac{dP}{dt} = \frac{d}{dt}\left[P(g(t))\right] = \frac{dP}{dg}\cdot\frac{dg}{dt}$$

$= 25 \cdot 30 = 750$

$$\frac{d}{dx}\left[(x^2+5x)^7\right] = 7(x^2+5x)^6(2x+5) = \frac{dP}{dg}\cdot\frac{dg}{dx}$$

$P(g) = g^7 = P(g(x)) = g(x)^7$

$g(x) = x^2+5x$

$$\frac{d}{dx}\left[g(x)^7\right] = 7g(x)^6\cdot\frac{dg}{dx}$$

$$\lim_{x\to 5} \frac{\sin(x-5)}{x^2+2x-35} = \lim_{x\to 5} \frac{\sin(x-5)}{(x-5)(x+7)}$$

$$= \lim_{x\to 5}\left(\frac{\sin(x-5)}{x-5}\right)\lim_{x\to 5}\left(\frac{1}{x+7}\right) = \frac{1}{12}$$

$$\lim_{x\to 5} \frac{\sin(x-5)}{x-5} = \lim_{u\to 0} \frac{\sin(u)}{u} = 1$$

L'HOPITAL'S RULE:

$$\lim_{x\to 5}\left(\frac{\sin(x-5)}{x^2+2x-35}\right) = \frac{0}{0} \text{ so take } \frac{\frac{d}{dx}[\text{top}]}{\frac{d}{dx}[\text{Bottom}]}$$

$$= \lim_{x\to 5} \frac{\cos(x-5)}{2x+2} = \frac{1}{12}$$

$$\lim_{x\to 2} \frac{(x-2)(x+5)}{(x-2)(x+7)} = \lim_{x\to 2} \frac{x^2+3x-10}{x^2+5x-14} = \lim_{x\to 2} \frac{2x+3}{2x+5} = \frac{7}{9}$$

$$= \lim_{x\to 2} \frac{x+5}{x+7} = \frac{7}{9}$$