Evaluate

$$\lim f = \lim_{x\to 1} \frac{x^{1000}-1}{x-1} = \lim_{x\to 1} \frac{(x-1)\overbrace{(x^{999}+x^{998}+\dots+x^2+x+1)}^{\text{cyclotomic polynomial}}}{x-1}$$

$$= \lim_{x\to 1} (x^{999}+x^{998}+\dots+x^2+x+1)$$

$$= \boxed{1000 = \lim f}$$

87. (a) Use the Product Rule twice to prove that if f, g, and h are differentiable, then $(fgh)' = f'gh + fg'h + fgh'$.

(b) Taking $f = g = h$ in part (a), show that

$$\frac{d}{dx}[f(x)]^3 = 3[f(x)]^2 f'(x)$$

(c) Use part (b) to differentiate $y = (x^4 + 3x^3 + 17x + 82)^3$.

(a) $(fgh)' = (f(gh))' = f'(gh) + f(gh)'$

$= f'gh + f(g'h + gh') = f'gh + fg'h + fgh'$

"Permutes where the prime goes."

(b) Use (a) to show

$(f^3)' = 3f^2f'$!

$(fff)' = f'ff + ff'f + fff'$

$= f^2f' + f^2f' + f^2f'$

$= 3f^2f'$

(c) **Use (b) to differentiate** $(x^4+3x^3+17x+82)^3$

$$\frac{d}{dx}\left[(x^4+3x^3+17x+82)^3\right] = \frac{d}{dx}\left[u^3\right] = 3u^2 \cdot \frac{du}{dx}$$

2.5

$$= 3(x^4+3x^3+17x+82)^2(4x^3+9x^2+17)$$

$$= 3f^2f'$$

This is your intro to the Chain Rule in Section 2.5!

$$\frac{d}{dx}\left[f(g(x))\right] = \frac{df}{dg}\cdot\frac{dg}{dx}$$

2.5

$$\frac{d}{dx}\left[f(u)\right] = \frac{df}{du}\cdot\frac{du}{dx}$$

$$\frac{d}{dx}\left[\sin(x^7)\right] = \boxed{\cos(x^7)\cdot 7x^6}$$

$$\frac{d}{dx}\left[\sqrt{\sin(x)+x^3}\right] = \frac{d}{dx}\left[(\sin(x)+x^3)^{\frac{1}{2}}\right]$$

$$= \boxed{\frac{1}{2}(\sin(x)+x^3)^{-\frac{1}{2}}(\cos(x)+3x^2)}$$

70. Suppose that $f(4) = 2$, $g(4) = 5$, $f'(4) = 6$, and $g'(4) = -3$. Find $h'(4)$.

(a) $h(x) = 3f(x) + 8g(x)$ (b) $h(x) = f(x)g(x)$

(c) $h(x) = \dfrac{f(x)}{g(x)}$ (d) $h(x) = \dfrac{g(x)}{f(x) + g(x)}$

2.3

70 $f(4)=2$, $g(4)=5$, $f'(4)=6$ & $g'(4)=-3$. we find $h'(4)$.

(a) $h(x)=3f(x)+8g(x) \longrightarrow$

$h'(x)=3f'(x)+8g'(x) \longrightarrow$

$h'(4)=3f'(4)+8g'(4)= 3(2)+8(-3) = \boxed{-18=h'(4)}$

(b) $h(x)= f(x)g(x) \longrightarrow$

$h'(4)= f'(4)g(4) + f(4)g'(4)$

$= (6)(5)+2(-3)$

$= 30-6 = \boxed{24 = h'(4)}$

$(fg)'=f'g+fg'$

$\left(\dfrac{f}{g}\right)'= \dfrac{f'g-fg'}{g^2}$

2.3 Differentiate:

$$\frac{d}{dx}\left[\frac{x}{x+\frac{c}{x}}\right] = \frac{d}{dx}\left[\frac{x}{x+cx^{-1}}\right] = \left(\frac{f}{g}\right)' = \frac{f'g-fg'}{g^2}$$

$$= \frac{1(x+cx^{-1}) - x(1-cx^{-2})}{(x+\frac{c}{x})^2}$$ ok for test. Let's see

if we can simplify:

1
1 1
1 2 1
1 3 3 1

$$= \frac{x+\frac{c}{x}-x+cx^{-1}}{(x+\frac{c}{x})^2}$$

$$= \frac{\frac{x^2+c-x^2+c}{x}}{x^2+2x\cdot\frac{c}{x}+\frac{c^2}{x^2}} = \frac{\frac{2c}{x}}{x^2+2c+\frac{c^2}{x^2}} = \frac{\frac{2c}{x}}{\frac{x^4+2x^2+c^2}{x^2}}$$

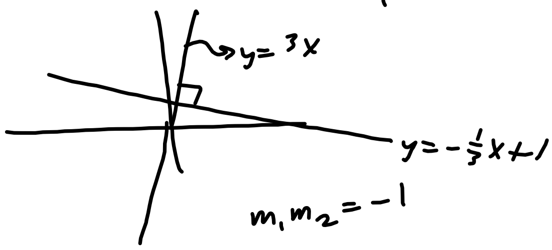
$$= \left(\frac{2c}{x}\right)\left(\frac{x^2}{x^4+2x^2+c^2}\right) = \frac{2cx}{x(x^4+2x^2+c^2)} = \frac{2cx}{x^5+2x^3+c^2x}$$

55–58 Find equations of the tangent line and normal line to the curve at the given point.

55. $y = x + \sqrt{x}$, $(1, 2)$ **56.** $y^2 = x^3$, $(1, 1)$

normal means perpendicular or orthogonal or right angle.

$m_2 = -\frac{1}{m_1}$



$m_1 m_2 = -1$

55 $y' = 1 + \frac{1}{2\sqrt{x}} \Rightarrow y'(1) = 1 + \frac{1}{2} = \boxed{\frac{3}{2} = m}$

$\frac{d}{dx}\left[\sqrt{x}\right] = \frac{d}{dx}\left[x^{\frac{1}{2}}\right] = \frac{1}{2}x^{-\frac{1}{2}}$

$L_1(x)$ = **tangent line at $x = 1$**

$= f'(1)(x-1) + f(1)$

$$\boxed{y = \frac{3}{2}(x-1) + 2}$$

$(1, 2) = (1, f(1))$

NORMAL LINE: $\boxed{y = -\frac{2}{3}(x-1) + 2}$

#56 $y^2 = x^3$ isn't $y = f(x)$ at all!

M1 $y = \pm\sqrt{x^3} = \pm x^{\frac{3}{2}}$

& $(1,1)$ on it $\Rightarrow$ we're on the $y = x^{\frac{3}{2}}$ branch.

$f(x) = x^{\frac{3}{2}}$ @ $x = 1$.

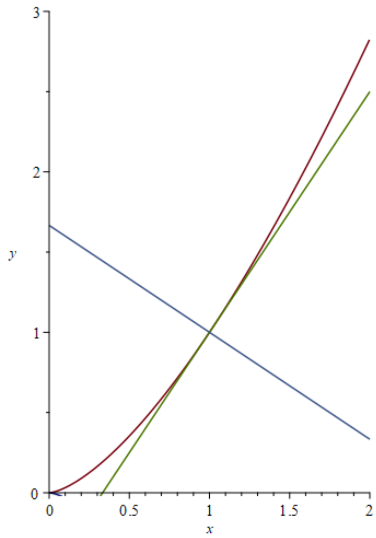
$\Rightarrow f'(x) = \frac{3}{2}x^{\frac{1}{2}}$

$\Rightarrow f'(1) = \frac{3}{2}$

$y = f'(1)(x-1) + f(1)$

$= \frac{3}{2}(x-1) + 1$ = tangent

& $y = -\frac{2}{3}(x-1) + 1$ = normal



§2.5 for dealing with the y^2.

$y^2 = x^3 \Longrightarrow \frac{d}{dx}$ [BOTH]

$\frac{d}{dx}[y^2] = 2y \cdot \frac{dy}{dx}$ by Chain Rule.

$= 2yy'$

$2yy' = 3x^2 \Longrightarrow$

$y' = \frac{3x^2}{2y}$

So, @ (1,1), we have

$y' = \frac{3}{2} \Longrightarrow$

tan line is

$\frac{3}{2}(x-1)+1$ &

normal line is

$-\frac{2}{3}(x-1)+1$