$x^n - a^n$ Divide by $x-a$:

$$\begin{array}{r|rrrrr} a & 1 & 0 & 0 & 0 \cdots & -a^n \\ & & a & a^2 & a^3 \cdots & a^n \\ \hline & 1 & a & a^2 & a^3 \cdot & 0 \end{array}$$

$$\frac{d}{dx}\left[x^n\right] = n x^{n-1}$$

This says $x^n - a^n = (x-a)(x^{n-1} + a x^{n-2} + a^2 x^{n-3} + \cdot + a^{n-1} x^0)$

$$\frac{f(x)-f(a)}{x-a} = \frac{x^n - a^n}{x-a} = \frac{(x-a)(x^{n-1} + a x^{n-2} + a^2 x^{n-3} + \cdots + a^{n-1})}{x-a}$$

$$= x^{n-1} + a x^{n-2} + a^2 x^{n-3} + \cdots + a^{n-1}$$

$x \neq a$

$$\xrightarrow{a \to x} \underbrace{x^{n-1} + x \cdot x^{n-2} + x^2 x^{n-3} + \cdots + x^{n-1}}_{n \text{ of them}}$$

$$= n x^{n-1}$$

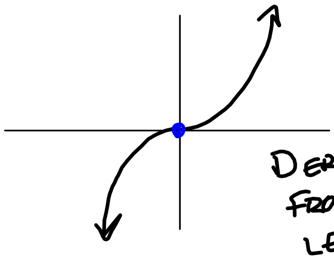
14c and 16 on 2.2?

Sketch x | x |

Where's it differentiable?

What's the formula for f'

$$f(x) = \begin{cases} x\cdot x & \text{if } x \ge 0 \\ x\cdot(-x) & \text{if } x<0 \end{cases} = \begin{cases} x^2 & \text{if } x\ge 0 \\ -x^2 & \text{if } x<0 \end{cases}$$ is cnt^s on $\mathbb{R}$.



DERIVATIVE FROM THE LEFT.

what is it @ x=0?

$$f_-'(0) = \lim_{h\to 0^-} \frac{f(0+h)-f(0)}{h}:$$

$$\frac{-(0+h)^2 - 0}{h} = \frac{-h^2}{h} = -h \xrightarrow{h\to 0^-} 0 \quad (h\neq 0)$$

$$f_-'(0) = \lim_{h\to 0^-} \frac{f(0+h)-f(0)}{h}$$

$$= \lim_{h\to 0^-} \frac{-h^2-0}{h} = \lim_{h\to 0^-} -h$$

$$= -\lim_{h\to 0^-} h = 0$$

$$f_+'(0) = \lim_{h\to 0^+} \frac{f(0+h)-f(0)}{h} = \lim_{h\to 0^+} \frac{h^2-0}{h} = \lim_{h\to 0^+} h = 0$$

$$f'(x) = \begin{cases} 2x & \text{if } x \ge 0 \\ -2x & \text{if } x<0 \end{cases} = |2x| = 2|x|$$

To get this w/ §2.2 skills:

$$x\ge 0 \quad \lim_{h\to 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h\to 0} \frac{x^2+2xh+h^2-x^2}{h} = \lim_{h\to 0}\left(\frac{2xh+h^2}{x}\right)$$

$$= \lim_{h\to 0} \frac{h(2x+h)}{h} = \lim_{h\to 0}(2x+h) = 2x$$

$x<0$: Same

$$\frac{f(14)-f(21)}{14-21} = \frac{40-55}{-7} \text{ etc.}$$ estimate for $H'(14)$

$$\cancel{\frac{f(21)-f(14)}{21-14}} =$$

Average these 2 for $H'(21)$

$$\frac{f(21)-f(28)}{21-28}$$

AVERAGE THESE 2 for $H'(28$

$$\frac{f(28)-f(35)}{28-35}$$

$H'(35)$

$$\frac{f(35)-f(42)}{-7}$$

$H'(42)$

$$\frac{f(42)-f(49)}{-7}$$

$= H'(49)$ estimate

16 §2.2

$f(x) = x^2 - \sqrt{x} + 2$

Find $f'(x)$.

x^2 ✓

$\frac{d}{dx}[x^2] = 2x$, $\frac{d}{dx}[2] = 0$

All we need is $\frac{d}{dx}[\sqrt{x}]$

& add 'em up.

$$\frac{f(x+h) - f(x)}{h} = \left(\frac{\sqrt{x+h} - \sqrt{x}}{h}\right)\left(\frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}\right)$$

$$= \frac{x+h-h}{h(\sqrt{x+h} + \sqrt{x})} = \frac{h}{h(\sqrt{x+h} + \sqrt{x})} = \frac{1}{\sqrt{x+h} + \sqrt{x}} \quad (h \neq 0) \xrightarrow{h \to 0}$$

$$\frac{1}{\sqrt{x} + \sqrt{x}} = \frac{1}{2\sqrt{x}}, \text{ so } \boxed{f'(x) = 2x + \frac{1}{2\sqrt{x}}}$$

$$\frac{d}{dx}\left[x^2+\sqrt{x}-2\right]=\frac{d}{dx}\left[x^2\right]+\frac{d}{dx}\left[\sqrt{x}\right]-\frac{d}{dx}\left[2\right]$$

$$\frac{f(x+h)-f(x)}{h}=\frac{(x+h)^2+\sqrt{x+h}-2-\left(x^2+\sqrt{x}-2\right)}{h}$$

$$=\frac{x^2+2xh+h^2+\sqrt{x+h}-2-x^2-\sqrt{x}+2}{h}$$

$$=\frac{2xh+h^2+\sqrt{x+h}-\sqrt{x}}{h}=\frac{2xh}{h}+\frac{\sqrt{x+h}-\sqrt{x}}{h}$$

$$=\frac{h(2x+h)}{h}+\left(\frac{\sqrt{x+h}-\sqrt{x}}{h}\right)\left(\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}\right)\quad (h\neq 0)$$

$$=\ldots=2x+h+\frac{1}{\sqrt{x+h}+\sqrt{x}}\xrightarrow{h\to 0}2x+\frac{1}{2\sqrt{x}}$$

$$\frac{d}{dx}\left[x^4\right] = \lim_{c\to x} \frac{f(x)-f(c)}{x-c}$$

$$= \frac{x^4-c^4}{x-c} = \frac{(x^2-c^2)(x^2+c^2)}{x-c}$$

$$= \frac{(x-c)(x+c)(x^2+c^2)}{x-c} = x^3 + c^2x + c\,x^2 + c^3$$

$x \neq c$

$$= x^3 + cx^2 + c^2x + c^3 \xrightarrow{c\to x} \underbrace{x^3 + x^3 + x^3 + x^3}_{\text{4 of 'em}}$$

$$= 4x^3$$

FACT:

$$x^n - c^n$$
$$= (x-c)(x^{n-1} + cx^{n-2} + c^2x^{n-3} + \cdots + c^{n-2}x^1 + c^{n-1})$$

Proof: Divide $x^n - c^n$ by $x-c$

$$x-c\,\overline{\big)\,x^n + 0x^{n-1} + \cdots + 0x - c^n}$$

$$\begin{array}{r|cccccc} c & 1 & 0 & 0 & \cdots & 0 & -c^n \\ & & c & c^2 & \cdots & c^{n-1} & c^n \\ \hline & 1 & c & c^2 & \cdots & c^{n-1} & 0 \\ & x^{n-1} & x^{n-2} & & \cdots & c & \text{Rem} \end{array}$$

This gives

$$x^n - c^n = (x-c)(x^{n-1} + cx^{n-2} + c^2x^{n-3} + \cdots + c^{n-2}x + c^{n-1})$$

$$x^3 - y^3 = (x-y)(x^2 + xy + y^2)$$

$$x^4 - y^4 = (x-y)(x^3 + yx^2 + y^2x + y^3)$$

Product Rule

$$\frac{d}{dx}\left[f(x)g(x)\right] = \frac{df}{dx}\cdot g(x) + f(x)\cdot\frac{dg}{dx}$$

$$(fg)' = f'g + fg'$$

Proof

$$\frac{f(x+h)g(x+h) - f(x)g(x)}{h}$$

$$= \frac{f(x+h)g(x+h) - f(x)g(x+h) + f(x)g(x+h) - f(x)g(x)}{h}$$

$$= \frac{\left(f(x+h) - f(x)g(x+h)\right) + f(x)\left(g(x+h) - g(x)\right)}{h}$$

$$= \frac{f(x+h) - f(x)}{h} g(x+h) + f(x)\frac{g(x+h) - g(x)}{h}$$

$$\xrightarrow{h \to 0} f'(x)g(x) + f(x)g'(x)$$

Quotient Rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{\frac{df}{dx}\cdot g(x) - f(x)\cdot\frac{dg}{dx}}{g(x)^2}$$

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

---

b & c are constants, f & g are differentiable functions

$$\frac{d}{dx}[c] = 0, \quad \frac{d}{dx}[x^n] = n\,x^{n-1}, \quad \frac{d}{dx}[cx] = c$$

$$\frac{d}{dx}[c f(x) + b g(x)] = c\cdot\frac{df}{dx} + b\,\frac{dg}{dx}$$

$$(cf + bg)' = cf' + bg'$$

$$\frac{d}{dx}\left[\frac{x^2-5x}{x^4+2x+1}\right] = \frac{(2x-5)(x^4+2x+1)-(x^2-5x)(4x^3+2)}{(x^4+2x+1)^2}$$

$$\left(\frac{f}{g}\right)' = \frac{f'g-fg'}{g^2}$$

$$f = x^2-5x \Longrightarrow f' = 2x' - 5$$

$$g = x^4+2x+1 \Longrightarrow 4x^3+2$$

2.4 Derivative of trig functions.

This will give the quotient an upper bound of 1.

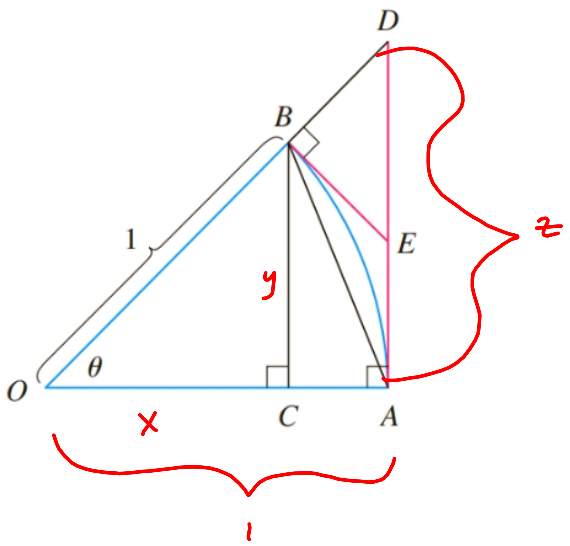
$$\sin\theta < \theta$$

This will make the quotient bigger than cosine.

$$\theta < \tan\theta$$

Then we can squeeze it in the limit:

$$\cos\theta < \frac{\sin\theta}{\theta} < 1$$



2.5 - The Chain Rule

$$\frac{d}{dx}\left[f(g(x))\right] = \frac{df}{dg}\cdot\frac{dg}{dx}$$

$$\frac{d}{dx}\left[\sqrt{x^5-7x}\right] = \frac{d}{dx}\left[(x^5-7x)^{\frac{1}{2}}\right] = \frac{1}{2}(x^5-7x)^{-\frac{1}{2}}(5x^4-7)$$

$$f(g) = g^{\frac{1}{2}} \qquad \frac{df}{dg} = \frac{1}{2}g^{-\frac{1}{2}} \qquad = \frac{5x^4-7}{2\sqrt{x^5-7x}}$$

$$g(x) = x^5-7x \qquad \frac{dg}{dx} = 5x^4-7$$