2.1 stuff

3. Consider the curve $f(x) = x - x^3$.

(a) Find the slope of the tangent line to the curve at the point (1, 0).

(b) Find an equation of the tangent line in part (a).

(c) Graph the curve and the tangent line.

2.1 $\frac{f(2+h)-f(2)}{h} \xrightarrow{h \to 0} f'(2)$

$\frac{f(x)-f(2)}{x-2} \xrightarrow{x \to 2} f'(2)$

& $f'(3)$ is a whole separate $\frac{f(3+h)-f(3)}{h}$ or $\frac{f(x)-f(3)}{x-3} \xrightarrow{x\to 3} f'(3)$

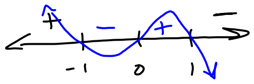
2.2 says $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{c \to x} \frac{f(x)-f(c)}{x-c}$

$= \lim_{c\to x} \frac{f(c)-f(x)}{c-x}$

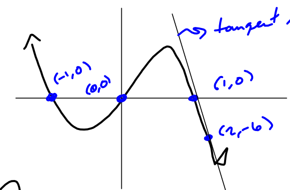
Then just evaluate $f'(2)$ & $f'(3)$ using $f'(x)$.

$f(x) = x - x^3 = -x^3 + x$ ↖ $-x^3$ ... ↘

$= -x(x^2-1) = -x(x+1)(x-1)$



$-x(x-1)(x+1)$



→ tangent line @ x=2 (wrong x, but whatever)

2.1 version: $\frac{f(x)-f(2)}{x-2} = \frac{x-x^3-(2-2^3)}{x-2}$ Difference of cubes.

$= \frac{x-2-x^3+2^3}{x-2} = \frac{x-2-(x^3-2^3)}{x-2} = \frac{(x-2)-(x-2)(x^2+2x+4)}{x-2}$

$= \frac{(x-2)[1-(x^2+2x+4)]}{(x-2)} = 1-x^2-2x-4 = -x^2-2x-3 \xrightarrow{x\to 2}$

$(x \neq 2)$

$-2^2-2(2)-3 = -4-4-3 = -11 = f'(1)$

$y - y_1 = m(x-x_1)$ $(x_1,y_1) = (1,0) = (1, f(1))$

$y = m(x-x_1)+y_1$ IDIOT JUST DID it for x=2

$= f'(2)(x-2)+f(2)$ $(2, 2-2^3) = (2,-6)$

$y = -11(x-2)-6$ is perfect.

Another 2.1 version

$\frac{f(2+h)-f(2)}{h} = \frac{2+h-(2+h)^3-(2-2^3)}{h}$

$= \frac{2+h-(2^3+3(2)^2(h)+3(2)(h)^2+h^3)+6}{h}$

$= \frac{2+h-8-12h-6h^2-h^3+6}{h}$

$= \frac{-11h-6h^2-h^3}{h} = \frac{h(-11-6h-h^2)}{h} = -11-6h-h^2 \xrightarrow{h\to 0} -11 = f'(2)$

$(h \neq 0)$

S2.2: $f'(x)$:

$\frac{f(x+h)-f(x)}{h} = \frac{x+h-(x+h)^3-(x-x^3)}{h}$ → You forgot me!

$= \frac{x+h-(x^3+3x^2h+3xh^2+h^3)-x+x^3}{h} = \frac{x+h-x^3-3x^2h-3xh^2-h^3-x+x^3}{h}$

$\frac{h-3x^2h-3xh^2-h^3}{h} = 1-3x^2-3xh-h^2 \xrightarrow{h\to 0} -3x^2+1$ ⟶

$h \neq 0$

$f'(2) = -3(2)^2+1$

$\boxed{-11 = f'(2)}$

S2.3: $f(x) = x - x^3$ ⟶

$f'(x) = 1-3x^2$ ⟶

$f'(2) = -11$

This one is in here to demonstrate Definition 1 and other ways of doing these.

4. Find an equation of the tangent line to the graph of f at the given point.

$$f(x) = \sqrt{x}, \quad (1, 1)$$

Using Definition 1:

$$m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a},$$

Show #5. They just want you to do these in as clumsy a way as possible.

Seriously, though, they're trying to get you to go from a constant, to an *a* = constant, but they're withholding just finding the slope for arbitrary *x*. I'm not sure why they belabor this so much.

#6 Find $f'(x)$ for $f(x) = mx+5$ 2.2 #6 Totally out of place.

$$\frac{f(x)-f(c)}{x-c} = \frac{mx+5-(mc+5)}{x-c} = \frac{mx+5-mc-5}{x-c}$$

$$= \frac{mx-mc}{x-c} = \frac{m(x-c)}{x-c} = m \xrightarrow{c \to x} \boxed{f'(x) = m}$$

$x \neq c$

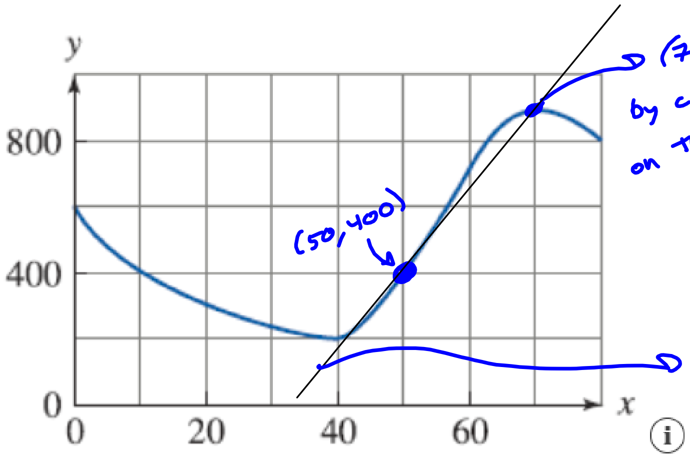
$$\frac{f(x+h)-f(x)}{h} = \frac{m(x+h)+5-mx-5}{h} = \frac{mx+mh-mx}{h}$$

$$= \frac{mh}{h} = m \xrightarrow{h \to 0} \boxed{m = f'(x)}$$

$h \neq 0$

Average Rate of Change of a function *f* on an interval [*a*, *b*].

8. The graph of a function *f* is shown.



(70, 900) by clicking on the graph.

Secant line

$m = \frac{900-400}{70-50} = \frac{500}{20} = \boxed{25 = m_{AVG}}$

(a) Find the average rate of change of *f* on the interval [50, 70].

25

[10, 50]

(b) Identify an interval on which the average rate of change of *f* is 0.

9. Find an equation of the tangent line to the graph of $y = g(x)$ at $x = 4$ if $g(4) = -3$ and $g'(4) = 5$. (Enter your answer as an equation in terms of y and x.)

$(x_1, y_1) = (4, -3) = (4, g(4))$

$m = 5$

$y = m(x - x_1) + y_1$

$= f'(4)(x-4) + f(4)$

$= g'(4)(x-4) + g(4)$

$= 5(x-4) - 3$

14. The limit represents the derivative of a function $y = f(x)$ at some number a. Find the function f and the value of a

$$\lim_{h \to 0} \frac{\cos(\pi + h) + 1}{h} = f'(c) \text{ for some } c.$$

Looks like $f(x) = \cos(x)$

$c = \pi$

$$\frac{\cos(\pi + h) - \cos(\pi)}{h} = \frac{\cos(\pi + h) + 1}{h} \xrightarrow{h \to 0} f'(\pi) \,!$$

16. The cost (in dollars) of producing x units of a certain commodity is $C(x) = 2{,}000 + 13x + 0.05x^2$.

(a) Find the average rate of change (in $ per unit) of C with respect to x when the production level is changed from $x = 100$ to the given value. (Round your answers to the nearest cent.)

(i) $x = 105$ 23.25 per unit

(ii) $x = 101$ 23.05 per unit

Let C = cost in dollars as a function of x = # of units produced of some commodity.

Lexicon: WORDS & UNITS for the variables used. Crucial for written work.

Given $C(x)$, we find

(i) $\dfrac{C(105) - C(100)}{105 - 100} = m_{AVG}$ on $[100, 105]$

$$= \frac{2000 + 13(105) + .05(105)^2 - (2000 + 13(100) + .05(100)^2)}{5}$$

$$= \frac{13(105) - 13(100) + .05(105)^2 - .05(100)^2}{5}$$

$$= \frac{13(5) + .05(105^2 - 100^2)}{5} = \frac{65 + .05(105-100)(105+100)}{5}$$

$$= \frac{65 + \frac{5}{100}(5)(205)}{5} = \frac{65 + (\frac{1}{20}(5)(205))}{5}$$

$$= \frac{65 + \frac{205}{4}}{5} = \frac{\frac{260 + 205}{4}}{5} = \frac{465}{4(5)} = \boxed{\frac{93.0000}{4}} = 23.25$$

(b) Find the instantaneous rate of change (in $ per unit) of C with respect to x when $x = 100$. (This is called the *marginal cost*. Its significance will be explained in a future chapter.)

23 per unit

$$C'(100) = \lim_{h \to 0} \frac{C(100+h) - C(100)}{h} \quad \ldots$$

18. The quantity (in pounds) of a gourmet ground coffee that is sold by a coffee company at a price of p dollars per pound is $Q = f(p)$.

(a) What is the meaning of the derivative $f'(5)$? What are its units?

(b) Is $f'(5)$ positive or negative? Explain.

$$f'(5) = \lim_{h \to 0} \frac{f(5+h) - f(5)}{h}$$

Let $Q = f(p)$

= Quantity (in pounds) of ground gourmet coffee as a function of

p = price of one pound of coffee $\left(\frac{\text{price}}{\text{lb}}\right)$ in \$/lb.

$$\frac{\text{pounds}}{\frac{\$}{\text{pound}}} = \frac{\text{pounds}^2}{\$}$$

Better than this

From the production side, you expect more quantity produced as the price rises, so $Q'(5)$ should be positive.

From the CUSTOMER'S point of view, less will be demanded as the price rises.

$$f(x) = \begin{cases} x\sin\left(\frac{2}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

Does $f'(0)$ exist?

#20

$$\frac{f(0+h)-f(0)}{h} = \frac{h\sin\left(\frac{2}{h}\right) - 0}{h}$$

$$= \underset{(h\neq 0)}{\sin\left(\frac{2}{h}\right)} \xrightarrow{h \to 0} \nexists$$

#21

$$\frac{f(0+h)-f(0)}{h} = \frac{h^2\sin\left(\frac{2}{h}\right) - 0}{h}$$

$$= \frac{h^2\sin\left(\frac{2}{h}\right)}{h} = \underset{(h\neq 0)}{h\sin\left(\frac{2}{h}\right)} \xrightarrow{h \to 0} \boxed{0 = f'(0)}$$

#21 $f(x) = \begin{cases} x^2\sin\left(\frac{\pi}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$

§2.2 stuff

#8 looks challenging.

9. Find the derivative of the function using the definition of derivative.

$f(x) = 3x^4$

Work this 2 ways:

$$\lim_{c \to x} \frac{f(x) - f(c)}{x - c}$$

$$\left(\lim_{c \to x} \frac{f(c) - f(x)}{c - x} \text{ is equivalent.}\right)$$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$