Questions?

Midterm News:

Scientific Calculator. No Graphing Calculator.

March 5th, before Spring Break. Covers Chapters 1 and 2.

Go to HORIZON HALL 103 to sign in (Ray Brown's office).

Probably be in Room 107 Suite D ish?

7 am - 8 pm are open times. I want to give you 2 hours on it.

If you can't make it, then, we'll make arrangements with the Testing Center. Franz will handle you.

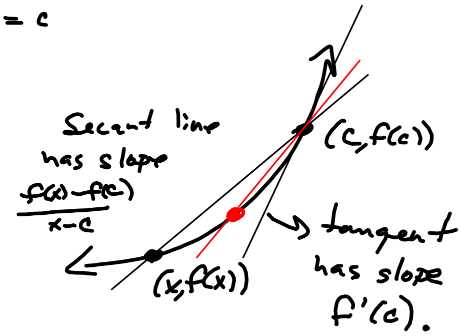
Sections 2.1, 2.2

Questions?

§2.1 Derivative at a point $x=c$

$$\frac{f(x)-f(c)}{x-c} \xrightarrow{x\to c} f'(c)$$

$$\frac{f(x)-f(2)}{x-2} \xrightarrow{x\to 2} f'(2)$$



§2.2 Derivative as a function.

$$\frac{f(x)-f(c)}{x-c} \xrightarrow{c\to x} f'(x)$$

$$\frac{f(c)-f(x)}{c-x} \xrightarrow{c\to x} f'(x)$$

$c=x+h \longrightarrow$

$$\frac{f(x+h)-f(x)}{h} \xrightarrow{h\to 0} f'(x)$$ Do this. plug in 2 & you get this

$$\frac{f(2+h)-f(2)}{h} \xrightarrow{h\to 0} f'(2)$$ Yucky

LIEBNIZ NOTATION

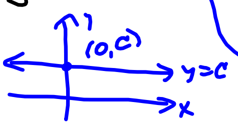
§2.3 Differentiation FORMULAS:

$$\frac{d}{dx}[f(x)] = \frac{df}{dx} = f'(x)$$ Liebniz & prime notation.

derivative with respect to x of f(x)

derivative of f w.r.t. x

$7 = 7x^0$ $\quad \frac{d}{dx}[\text{constant}] = 0$



$$\frac{d}{dx}[x] = 1$$

$$\frac{d}{dx}[cf] = c\frac{d}{dx}[f]$$ $\quad \frac{d}{dx}[3x^5] = 3\cdot 5x^4 = 15x^4$

$$\frac{d}{dx}[x^n] = nx^{n-1} \quad \forall\, n \neq 0.$$ $\quad \frac{d}{dx}[x^5] = 5x^4$

(There's more to §2.3, but 3 things here that will allow you to check 2.1 & 2.2.)

NEXT TIME FROM 2.3: c,d constant, f,g differentiable:

$$\frac{d}{dx}[cf(x)+dg(x)] = c\frac{df}{dx} + d\frac{dg}{dx}$$

$$\frac{d}{dx}[3x^5+7x^{-3}] = 15x^4-21x^{-4}$$

$$\frac{d}{dx}[f(x)g(x)] = f'g + fg'$$ Product Rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'g-fg'}{g^2}$$ QUOTIENT Rule

$$\frac{d}{dx}[x^2(x^7)] = (2x)(x^7) + (x^2)(7x^6) = 9x^8$$

$$\frac{d}{dx}[x^9] = 9x^8$$

$$\frac{d}{dx}(\sqrt{x}) = \frac{d}{dx}[x^{\frac{1}{2}}] = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}!$$

2.2 # 13

2.1 version:

Find the slope of $f(x) = \sqrt[3]{x}$ at $x = 8$

$$\lim_{x \to 8} \frac{f(x) - f(8)}{x - 8}$$

Need sum/difference of 2 cubes

Sum:

$x^3 + y^3 = (x+y)(x^2 - xy + y^2)$

DIFFERENCE:

$x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

$(\sqrt[3]{x})^3 - (\sqrt[3]{8})^3 = (\sqrt[3]{x} - \sqrt[3]{8})\left((\sqrt[3]{x})^2 + \sqrt[3]{x} \cdot 8 + (\sqrt[3]{8})^2\right)$

$8 \to \sqrt[3]{8}$

$= x - 8!$

$$\frac{\sqrt[3]{x} - \sqrt[3]{8}}{x - 8}$$

$$= \frac{(\sqrt[3]{x} - \sqrt[3]{8})(\sqrt[3]{x^2} + 8\sqrt[3]{x} + \sqrt[3]{64})}{(x-8)(\sqrt[3]{x^2} + 8\sqrt[3]{x} + 4)}$$

$8 \to \sqrt[3]{8}$

$$= \frac{x - 8}{(x-8)(\sqrt[3]{x^2} + 8\sqrt[3]{x} + 4)} = \frac{1}{\sqrt[3]{x^2} + 8\sqrt[3]{x} + 4} \xrightarrow{x \to 8}$$

$8 \to 2$

$(x \neq 8)$

$$\frac{1}{\sqrt[3]{8^2} + 8\sqrt[3]{8} + 4}$$

$8 \to 2$

$$= \frac{1}{4 + 8 \cdot 2 + 4} = \frac{1}{24}$$

$8 \to 2$, $24 \to 12$

2.3 way: Find $f'(8)$ for $f(x) = \sqrt[3]{x} = x^{\frac{1}{3}}$

$\Longrightarrow f'(x) = \frac{1}{3} x^{-\frac{2}{3}} = \frac{1}{3 \cdot x^{\frac{2}{3}}} \Longrightarrow$

$f'(8) = \frac{1}{3 \cdot 8^{2/3}} = \frac{1}{3 \cdot 2^2} = \frac{1}{12}$ oops!

The trick was:

$x - 8 = (\sqrt[3]{x})^3 - 2^3 = a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

Find slope of $f(x) = \sqrt{x}$ @ $x = 4$

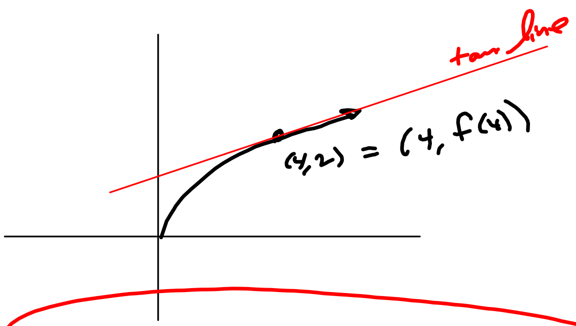
2.1

$$\frac{f(x) - f(4)}{x-4} \xrightarrow{x \to 4} f'(4)$$

$$\frac{f(4+h) - f(4)}{h} \xrightarrow{h \to 0} f'(4)$$

$$\frac{\sqrt{x} - \sqrt{4}}{x-4} = \frac{\sqrt{x} - 2}{x-4} = \frac{(\sqrt{x}-2)(\sqrt{x}+2)}{(x-4)(\sqrt{x}+2)}$$

$$= \frac{x-4}{(x-4)(\sqrt{x}+2)} = \frac{1}{\sqrt{x}+2} \xrightarrow{x \to 4} \frac{1}{4}$$



What's the tangent line's equation?

Recall:

$y - y_1 = m(x - x_1)$

$y = m(x - x_1) + y_1$

$y = f'(4)(x-4) + f(4)$

$$\boxed{y = \frac{1}{4}(x-4) + 2}$$

Perfect for written work.

Book likes

$y = f(4) + f'(4)(x-4)$

2.1 #1

A curve has equation $y = f(x)$.

(a) Write an expression for the slope of the secant line through the points $P(3, f(3))$ and $Q(x, f(x))$.

- $\dfrac{f(x) - x}{f(3) - 3}$
- $\dfrac{x - 3}{f(x) - f(3)}$
- $\dfrac{f(x) - f(3)}{x - 3}$
- $\dfrac{f(3) - 3}{f(x) - x}$

(b) Write an expression for the slope of the tangent line at P.

$$\lim_{x \to 3} \frac{f(x) - f(3)}{x - 3}$$

2.1 #2

Consider the parabola $y = 4x - x^2$.

(a) Find the slope of the tangent line to the parabola at the point (1, 3).

[ ] ✖ 2

(b) Find an equation of the tangent line in part (a).

$y =$ [ ] ✖ $2x + 1$

(c) Graph the parabola and the tangent line.

2.2 way $f(x)$'s tangent @ $x=1$

$f(x) = -x^2+4x \Longrightarrow$

$$\frac{f(x)-f(1)}{x-1} = \frac{-x^2+4x-(1^2+4(1))}{x-1} = \frac{-x^2+4x-3}{x-1}$$

$$\cancel{= \frac{(x+3)(x-1)}{x-1} = \underset{x\neq 1}{x+3} \xrightarrow{x\to 1} 4}$$

$$= \frac{-(x^2-4x+3)}{x-1}$$

$$= \frac{-(x-1)(x-3)}{x-1}$$

$$= \underset{x\neq 1}{-(x-3)} \xrightarrow{x\to 1} +2$$

FINALLY!

2.3 way

$f(x) = -x^2+4x \Longrightarrow$

$f'(x) = -2x+4 \Longrightarrow$

$f'(1) = -2(1)+4 = 2$

2.2 exercises

ven graph of $f(x)$ to sketch the graph of f'.