$$\lim_{x \to 0} x^3 \cos\left(\frac{\pi}{x}\right)$$

The $x^3 \to 0$ as $x \to 0$ &

the $\cos\left(\frac{\pi}{x}\right)$ is no bigger than 1.

so, in essance

$x^3 \cos\left(\frac{\pi}{x}\right) \to 0$ as $x \to 0$

because $x^3 \cdot 1 \to 0$ as $x \to 0$

DAMPENING that makes something converge that otherwise wouldn't.

$\cos\left(\frac{\pi}{x}\right)$ is problematic @ $x=0$.

$x^2 \cos\left(\frac{\pi}{x}\right)$ is damped @ $x=0$ so it can't misbehave beyond a certain amount in the limit.

Note $0^3 \cdot \cos\left(\frac{\pi}{0}\right)$ is undefined.

The function

$$\begin{cases} x^2 \cos\left(\frac{\pi}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$ is continuous!

CONTINUITY DEF'N: $\lim_{x \to c} f(x) = f(c)$

Prove $\lim_{x \to 0} x^2 \sin\left(\frac{\pi}{x}\right) = 0$.

Proof:

$$-1 \le \sin\left(\frac{\pi}{x}\right) \le 1 \quad \forall\, x \ne 0$$

$$-x^2 \le x^2 \sin\left(\frac{\pi}{x}\right) \le x^2$$

$$\begin{array}{ccccc} \downarrow{\scriptstyle x \to 0} & & \downarrow{\scriptstyle x \to 0} & & \downarrow{\scriptstyle x \to 0} \\ 0 & & 0 & & 0 \end{array}$$

i.e. $x^2 \sin\left(\frac{\pi}{x}\right) \xrightarrow{x \to 0} 0$.

Any questions, joseph?

Evaluate the difference quotient for the given function. Simplify your answer.

$f(x) = 1 + 4x - x^2, \quad \frac{f(3+h) - f(3)}{h}$

$-x^2+4x+1$

$\boxed{-h-2}$

$$\frac{f(3+h)-f(3)}{h} = \frac{1+4(3+h)-(3+h)^2 - \left[1+4(3)-3^2\right]}{h}$$

$f(x) = -x^2+4x+1$

$$\frac{f(x+h)-f(x)}{h} = \frac{-(x+h)^2+4(x+h)+1-\left[-x^2+4x+1\right]}{h}$$

Mon my speed!