I think I mushed on through Section 1.7, last week, and all that remains, there, is

Proving polynomial limits of higher degree.

This is the general polynomial case. Since we have cubic and quartic formulas, and the limiting value always allows us to split off a real factor, we can handle proofs up to degree *n* = 5, by analyzing the polynomial of degree *n* - 1 = 4, 3, 2, 1, or 0.

Anything above a linear polynomial is going to be bonus on the written portion of your homework and tests.

I will post Writing Project #1, today or tomorrow, in Assignments on D2L. Writing Project #0 should be up there, now.

You will see at least one higher-degree proof for bonus. I'll be pretty stringent, but the forms to follow, the quantifiers and the language used, are all very standard, and go the same way, every time.

$\lim_{x\to 2}(3x-5) = 1$ counts

Anything like $\lim_{x\to 1}(x^3+x+3) = 5$ is bonus

§1.7 #19 sucks.

Questions?

We can spend all day.

Otherwise, I'm going to do §1.7 higher-degree polynomial proofs. If you can do those, you really get it.

But people may still be wading through earlier sections and be primed for timely answers, live. We can go one of two or three or five directions.

#19 in Section 1.7 is a good one. I hate the way they wrote it, and I don't understand their answer - or I didn't - because they used a different font in the question than in the answer. Crappy epsilon.

$$\lim_{x \to 2} (3x-7) = -1$$

The divisor of ε is the slope (How fast y grows with respect to x.

<u>Proof</u>

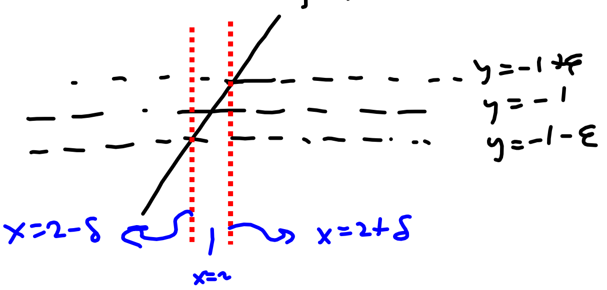
Let $\varepsilon > 0$. Define $\delta = \frac{\varepsilon}{3}$.

Then $0 < |x-2| < \delta \Longrightarrow$

$$|3x-7-(-1)| = |3x-6|$$

$$= 3|x-2| < 3\delta = 3 \cdot \frac{\varepsilon}{3}$$

$$= \varepsilon$$



<u>Claim:</u> $\lim_{x \to 2} (3x-7) = -1$

<u>Proof</u> Let $\varepsilon > 0$. Define $\delta = \frac{\varepsilon}{3}$. Then $0 < |x-2| < \delta$

$\Longrightarrow |3x-7-(-1)| = |3x-6| = 3|x-2| < 3\delta = 3 \cdot \frac{\varepsilon}{3} = \varepsilon$ ■

§1.7 #19 Charlie's version.

$$\lim_{x\to 1} (x^3+x+3) = 5$$

want $|x^3+x+3-5| < \varepsilon$

$|x^3+x-2| < \varepsilon$.

x^3+x-2 is guaranteed to have a factor of $x-1$ if we're right.

split off that factor:

$$\begin{array}{r|rrrr} 1 & 1 & 0 & 1 & -2 \\ & & 1 & 1 & 2 \\ \hline & 1 & 1 & 2 & 0 \end{array}$$

This says $x^3+x-2 = \underbrace{(x-1)}_{<\delta!}\underbrace{(x^2+x+2)}$

Need a ceiling on this for "x close to x=1."

Preview:

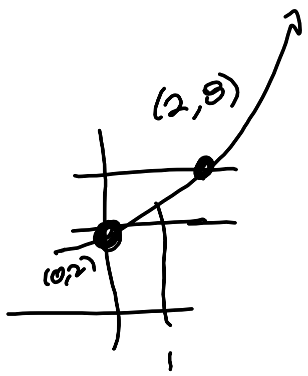
$|x^3+x+3-5| = |x^3+x-2|$

$= |x-1||x^2+x+2| < \delta|x^2+x+2|$

x^2+x+2 :

$x^2+x+\left(\frac{1}{2}\right)^2-\frac{1}{4}+2$

$=\left(x+\frac{1}{2}\right)^2+\frac{7}{4}$



Keep within $\delta \le 1$ unit of $x=1$

$2^2+2+2=8$ is what we need.

The max of x^2+x+2 on $[0,2]$.

Proof

Let $\varepsilon>0$ be given. Define $\delta=\min\{1,\frac{\varepsilon}{8}\}$. Then if $0<|x-1|<\delta$, we have $|x^3+x+3-5|=|x^3+x-2|$

$=|x-1||x^2+x+2|\le 8|x-1|<8\delta\le 8\cdot\frac{\varepsilon}{8}=\varepsilon$ ∎