Find two functions whose limits don't exist but their sum does.

$\lim_{x\to 0} \frac{1}{x}$ & $\lim_{x\to 0} -\frac{1}{x}$:

limits don't exist, but $\lim_{x\to 0}\left(\frac{1}{x}+\left(-\frac{1}{x}\right)\right)=\lim_{x\to 0}(0)=0.$

31. [–/2 Points] DETAILS SCALC9 1.6.067.

Is there a number a such that the following limit exists? (If an answer does not exist, enter DNE.)

$$\lim_{x\to -2} \frac{2x^2+ax+a+6}{x^2+x-2}$$

Find the value a.

$\frac{\text{stuff}}{x^2+x-2} = \frac{\sim}{(x+2)(x-1)}$

Want $x+2$ to be a factor of

$2x^2+ax+a+6$:

$x(2x+a) = x(2x+4)$

Ans: $a=14$?!

$2x^2+ax+a+6 = (x+2)(\text{something})$

$2x^2+4x-4\cdot x+ax+a+6$

$= 2x^2+4x-4x+ax+a+3x+6-3x$

$= 2x(x+2)+3(x+2)+ax+a-7x$

$=$ $ax+a-7x =$

$2x^2+ax+a+6 \xrightarrow{x\to -2} 0$ must be the case

Jesse saw this

$2(-2)^2+(-2)a+a+6=$

$= 8-2a+a+6=0 \Rightarrow$

$14-a=0 \Rightarrow$

$\boxed{14=a}$!

$2x^2+14x+14+6$

$= 2x^2+14x+20$

$= 2(x^2+7x+10)$

$= 2(x+5)(x+2)$

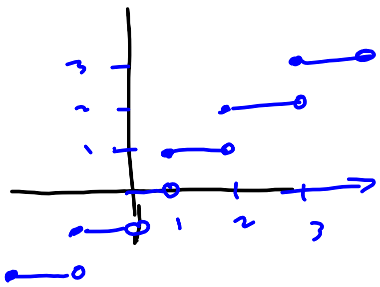
So $a=14 \Rightarrow$

$\frac{2x^2+ax+a+2}{x+2}$

$= \frac{2x^2+14x+20}{x+2}$

$= \frac{2(x+5)\cancel{(x+2)}}{\cancel{x+2}} = \underset{x\neq -2}{2(x+5)} \xrightarrow{x\to -2} \boxed{6}$

$[\![x]\!]$ = greatest integer $\leq x$.



11. A machinist is required to manufacture a circular metal disk with area 1000 cm^2.

(a) What radius produces such a disk?

(b) If the machinist is allowed an error tolerance of ± 5 cm^2 in the area of the disk, how close to the ideal radius in part (a) must the machinist control the radius?

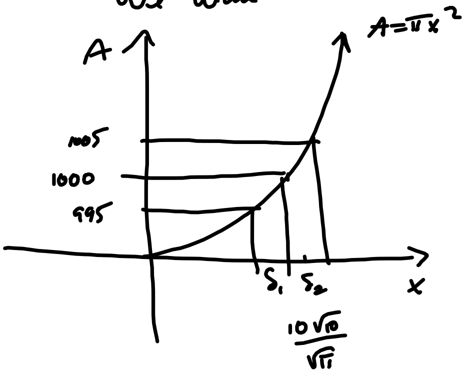
(c) In terms of the ε, δ definition of $\lim_{x \to a} f(x) = L$, what is x? What is $f(x)$? What is a? What is L? What value of ε is given? What is the corresponding value of δ?

(a) $\pi r^2 = 1000 \text{ cm}^2$

$r^2 = \frac{1000}{\pi}$

$r = \pm\sqrt{\frac{1000}{\pi}} \longrightarrow r = \frac{10\sqrt{10}}{\sqrt{\pi}}$

We want that number, $\pm 5 \text{ cm}^2$



$\delta_2 = \frac{\sqrt{1005} - 10\sqrt{10}}{\sqrt{\pi}} \approx$ 0.04454749

$\delta_1 = \frac{10\sqrt{10} - \sqrt{995}}{\sqrt{\pi}} \approx .04465900$

-0.04465900

$\frac{\sqrt{1005}}{\sqrt{\pi}}$ is closer to $\frac{10\sqrt{10}}{\sqrt{\pi}}$ than $\frac{\sqrt{995}}{\sqrt{\pi}}$ is, so, we use that as our tolerance in the x-direction

Use $\delta_2 = \frac{\sqrt{1005} - 10\sqrt{10}}{\sqrt{\pi}} \approx$ 0.04454749

as it's the smaller of the 2.

If we're within .0445479, we're close enough on <u>both</u> sides of $\frac{10\sqrt{10}}{\sqrt{\pi}}$

§1.7: ε = hoped-for area tolerance (output variable)

δ = tolerance we set on the input variable

How close to a particular *x* do we have to be in order to keep *f*(*x*) sufficiently close to a particular *y*-value?

Want δ such that if $|x-a|<\delta$, we have $|f(x)-L|<\varepsilon$.

Claim:
$\lim_{x\to 3}(2x+7)=13$.

Proof:
Let $\varepsilon>0$ be given.

Scratch:
want $|2x+7-13|<\varepsilon$

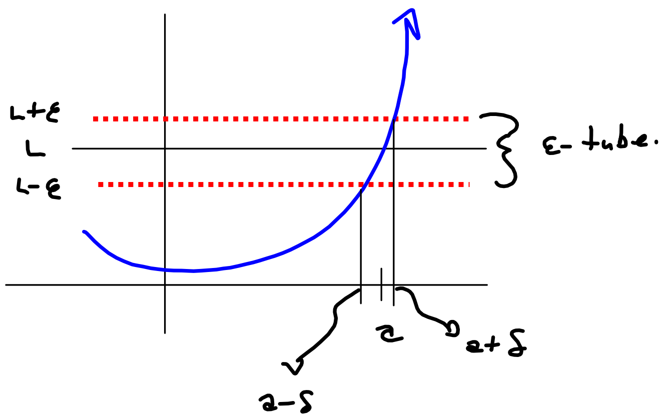
$\iff |2x-6|<\varepsilon$

$\iff 2|x-3|<\varepsilon$

$|x-3|<\frac{\varepsilon}{2}$

Define $\delta=\frac{\varepsilon}{2}$. Then $0<|x-3|<\frac{\varepsilon}{2} \implies$

$|2x+7-13|=|2x-6|=2|x-3|<2\cdot\frac{\varepsilon}{2}=\varepsilon$ ∎



That's the extent of what you'll be tested on under a time control (except for bonus).

Claim: $\lim_{x \to 5} (3x-2) = 13$

Proof

Let $\varepsilon > 0$ be given. Define $\delta = \frac{\varepsilon}{3}$ so that if $0 < |x-5| < \delta$, we have $|3x-2-13| = |3x-15| = 3|x-5| < 3 \cdot \frac{\varepsilon}{3} = \varepsilon$ ∎

Claim: $\lim_{x\to 2}(x^2-5x+7) = 1$

Scratch 1^{st}, assume $\delta \le 1$, i.e., $|x-2| \le 1$.

want $|x^2-5x+7-1| = |x^2-5x+6|$

$= \underbrace{|x-3|}_{<?}\underbrace{|x-2|}_{<\delta} < \varepsilon$

Now $\delta \le 1$ means $1 \le x \le 3$

$\Rightarrow 1-3 \le x-3 \le 0$, i.e.

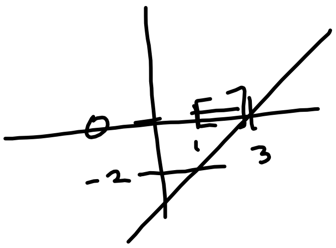
$-2 \le x-3 \le 0$

How bad can $|x-3|$ get on $[1, 3]$?

No worse than 2.

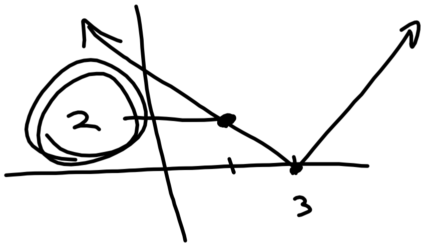
$-2 \le x-3$ means

$-(-2) \ge -(x-3)$



what's the biggest this gets in absolute value?

$|x-3|$



So $|x-3| \le 2$ if $1 \le 2 \le 3$

<u>Proof</u>

Let $\epsilon > 0$ be given. Define $\delta = \min\{1, \frac{\epsilon}{2}\}$.

Then $0 < |x-2| < \epsilon \implies |x^2 - 5x + 7 - 1|$

$= |x^2 - 5x + 6| = |x-3||x-2| \le 2|x-2| < 2 \cdot \frac{\epsilon}{2} = \epsilon$ ■