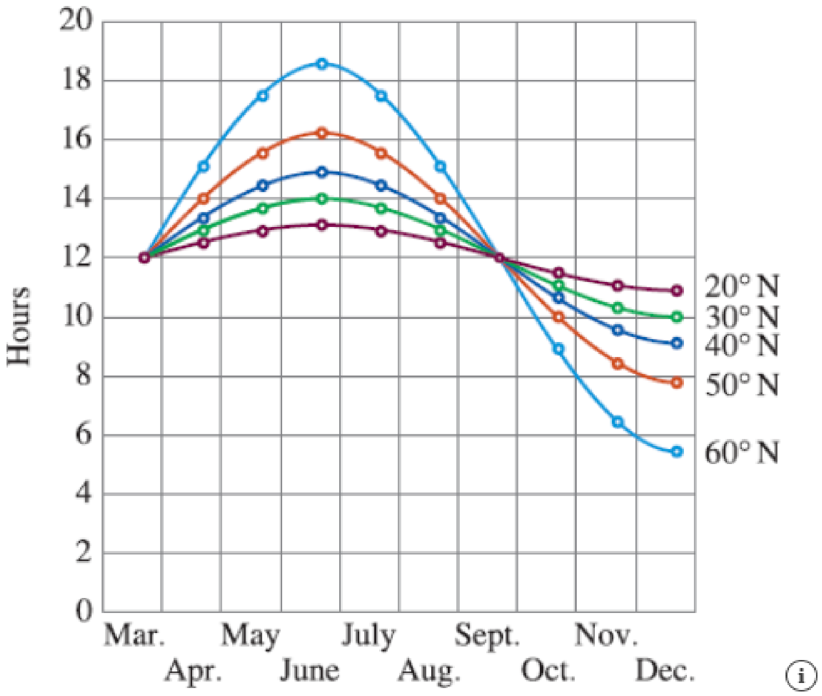
† Graph of the length of daylight from March 21 through December 21 at various latitudes.



$L(t) = \boxed{2\sin\left(\frac{2\pi}{365}(t-80)\right)+12}$ ✓ $\boxed{2\sin\left(\frac{2}{365}\pi(t-80)\right)+12}$

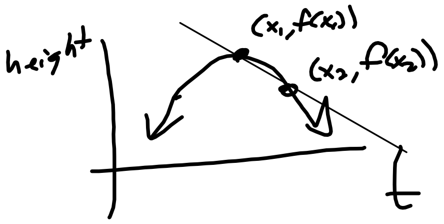
31, 28, 21

$31+28+21=80$, so

3/21 is $t=80$.

1.4 #3

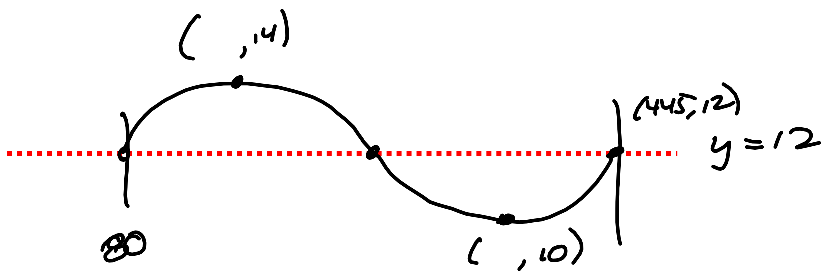
Avg speed is slope



Hours in varied latitudes

	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
30°	12	13.2	13.7	14	13.8	12.8	12	11.2	10.2	10

T = period = 365 days
start @ midline of y = 12 hrs of daylight.
Use sine that starts @ t = ~~80th~~ day.



365 + 80 = 445

$a\sin(b(x-c)) + d$

$= 2\sin(b(x-80)) + 12$

Period is 365, so
$bx = 2\pi$ when $x = 365$ ⟶
$365b = 2\pi$

$b = \frac{2\pi}{365}$

$= 2\sin\left(\frac{2\pi}{365}(x-80)\right) + 12$ = HOURS / DAY of sunlight, using March 21st as 1st day.

1.5 - Limit of a Function

1 Intuitive Definition of a Limit Suppose $f(x)$ is defined when x is near the number a. (This means that f is defined on some open interval that contains a, except possibly at a itself.) Then we write

$$\lim_{x \to a} f(x) = L$$

and say "the limit of $f(x)$, as x approaches a, equals L"

if we can make the values of $f(x)$ arbitrarily close to L (as close to L as we like) by restricting x to be sufficiently close to a (on either side of a) but not equal to a.

Challenge: make $|f(x) - L|$ small by making $|x-a|$ small enough!

The limit of the difference quotient is the limit in which we're most interested, and it always takes the form of a 0/0 situation!

Book Example:

$$\lim_{t \to 0} \frac{\sqrt{t^2+9} - 3}{t^2}$$

$$(a-b)(a+b) = a^2 - b^2$$

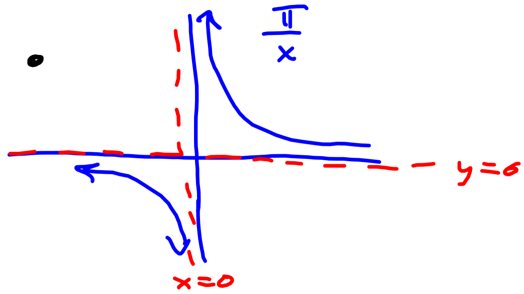
$$\frac{\sqrt{t^2+9} - 3}{t^2} = \frac{(\sqrt{t^2+9} - 3)(\sqrt{t^2+9} + 3)}{t^2(\sqrt{t^2+9} + 3)}$$

$$= \frac{t^2+9-9}{t^2(\sqrt{t^2+9}+3)} = \frac{\cancel{t^2}}{\cancel{t^2}(\sqrt{t^2+9}+3)} = \frac{1}{\sqrt{t^2+9}+3} \quad (t \neq 0)$$

$$\xrightarrow{t \to 0} \frac{1}{\sqrt{9}+3} = \frac{1}{3+3} = \boxed{\frac{1}{6}}$$

Example 4 $f(x) = \sin\left(\frac{\pi}{x}\right)$

$\lim_{x \to 0} f(x) =$



Using just 0.1, 0.01, 0.001, ...

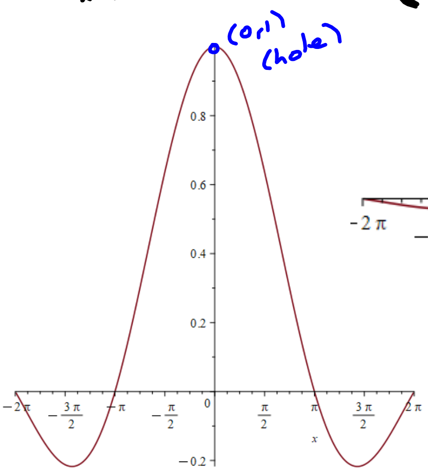
$$\left.\begin{array}{l} \dfrac{\pi}{\frac{1}{10}} = 10\pi \\ \dfrac{\pi}{\frac{1}{1000}} = 1000\pi \end{array}\right\} \text{Integer multiples of } \pi.$$

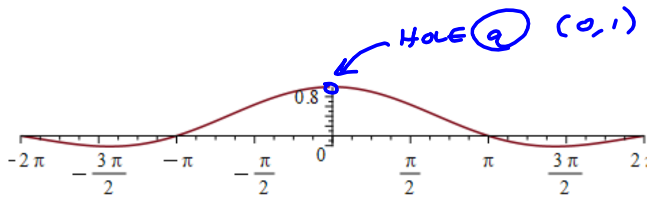
$\sin(n\pi) = 0.$

So the limit is zero if you are too naive. So beware of numerical methods....

EXAMPLE 5 $f(x) = \frac{\sin(x)}{x}$ IMPORTANT! REMEMBER! (S2.4!)

$\lim_{x \to 0} \frac{\sin(x)}{x} = 1$ (Graphical confirmation w/ CAS (Maple))





2 **Definition of One-Sided Limits** We write

$$\lim_{x \to a^-} f(x) = L$$

and say the **left-hand limit of $f(x)$ as x approaches a** [or the **limit of $f(x)$ as x approaches a from the left**] is equal to L if we can make the values of $f(x)$ arbitrarily close to L by taking x to be sufficiently close to a with x *less than* a.

Same thing for right-handed limits. Just turn the "-" into a "+" and replace "left" with "right." $\lim_{x \to a^+} f(x) = L$

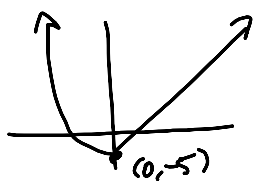
The "regular" limit exists if and only if the left and right limits both exist and both agree.

3 $\lim_{x \to a} f(x) = L$ if and only if $\lim_{x \to a^-} f(x) = L$ and $\lim_{x \to a^+} f(x) = L$

$$\lim_{x \to 3^-} (x^2-5) = 3^2-5 = \boxed{4}$$

$$\lim_{x \to 3^+} (x^2-5) = \text{same.}$$

$$f(x) = \begin{cases} x^2-5 & \text{if } x < 0 \\ 2x-5 & \text{if } x \geq 0 \end{cases}$$



$$\left.\begin{aligned} \lim_{x \to 0^-} f(x) &= 0^2-5 = -5 \\ \lim_{x \to 0^+} f(x) &= 2(0)-5 = -5 \end{aligned}\right\}$$

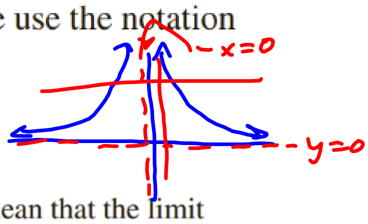
$$\therefore \lim_{x \to 0} f(x) = -5 \quad \blacksquare$$

Infinite Limits

EXAMPLE 8 Find $\lim_{x \to 0} \frac{1}{x^2}$ if it exists.

To indicate the kind of behavior exhibited in Example 8, we use the notation

$$\lim_{x \to 0} \frac{1}{x^2} = \infty$$



This does not mean that we are regarding ∞ as a number. Nor does it mean that the limit exists. It simply expresses the particular way in which the limit does not exist: $1/x^2$ can be made as large as we like by taking x close enough to 0.

Give me an $M > 0$
I can make $f(x) > M$ by
choosing x close enough to a.

In general, we write symbolically

$$\lim_{x \to a} f(x) = \infty$$

to indicate that the values of $f(x)$ tend to become larger and larger (or "increase without bound") as x becomes closer and closer to a.

4 Intuitive Definition of an Infinite Limit Let f be a function defined on both sides of a, except possibly at a itself. Then

$$\lim_{x \to a} f(x) = \infty$$

means that the values of $f(x)$ can be made arbitrarily large (as large as we please) by taking x sufficiently close to a, but not equal to a.

5 Definition Let f be a function defined on both sides of a, except possibly at a itself. Then

$$\lim_{x \to a} f(x) = -\infty$$

means that the values of $f(x)$ can be made arbitrarily large negative by taking x sufficiently close to a, but not equal to a.

The symbol $\lim_{x \to a} f(x) = -\infty$ can be read as "the limit of $f(x)$, as x approaches a, is negative infinity" or "$f(x)$ decreases without bound as x approaches a." As an example we have

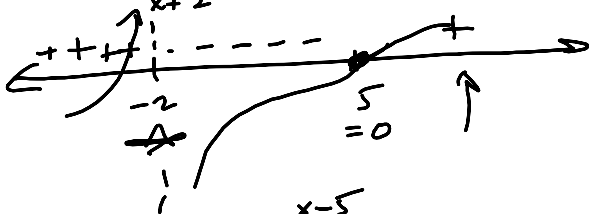
$$\lim_{x \to 0} \left(-\frac{1}{x^2} \right) = -\infty$$

Useful for a graph, but strictly speaking, these infinite limits do not exist as real numbers.

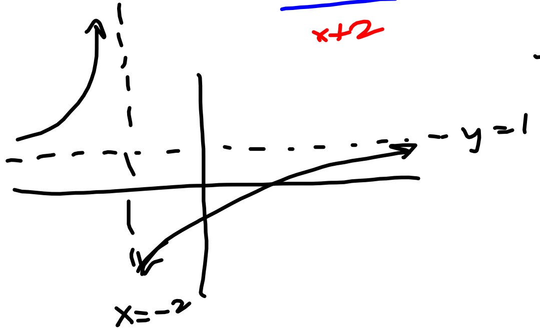
These infinite limits correspond to vertical asymptotes in graphs.

Sketch the graph of a simple linear/linear rational function.

$f(x) = \dfrac{x-5}{x+2}$ cut points: $x = -2, 5$



$\dfrac{x-5}{x+2}$



$\lim_{x \to -2^-} f(x) = +\infty$

$\lim_{x \to -2^+} f(x) = -\infty$

Section 1.6 - Limit Laws

Limit Laws Suppose that c is a constant and the limits

$$\lim_{x \to a} f(x) \qquad \text{and} \qquad \lim_{x \to a} g(x)$$

exist. Then

1. $\lim_{x \to a} [f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$

2. $\lim_{x \to a} [f(x) - g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$

3. $\lim_{x \to a} [cf(x)] = c \lim_{x \to a} f(x)$

4. $\lim_{x \to a} [f(x)g(x)] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$

5. $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$ if $\lim_{x \to a} g(x) \neq 0$

$\mathcal{S}$ $c \in \mathbb{R}$, $\lim_{x \to a} f(x) = L$, $\lim_{x \to a} g(x) = M$. Then

$$\lim (f \pm g) = L \pm M$$
$$\lim cf = cL$$
$$\lim (fg) = LM$$
$$\lim \left(\frac{f}{g}\right) = \frac{L}{M}, \text{ if } M \neq 0.$$
$$\lim (f^n) = L^n \quad \forall\, n \in \mathbb{N}$$
$$\lim (c) = c$$
$$\lim (x) = a$$
$$\lim (x^n) = a^n \quad \forall\, n \in \mathbb{N}$$
$$\lim (\sqrt[n]{f}) = \sqrt[n]{L} \quad \forall n \in \mathbb{N}$$

If $n = 2m$ for $m \in \mathbb{N}$, then assume $L > 0$.

Let's speed the evaluations up:

Direct Substitution Property If f is a polynomial or a rational function and a is in the domain of f, then

$$\lim_{x \to a} f(x) = f(a)$$

If $f(x) = g(x)$ when $x \neq a$, then $\lim_{x \to a} f(x) = \lim_{x \to a} g(x)$, provided the limits exist.

This is why there's a string of equal signs from the original difference quotient to the final passage to the limit at the end, after you cancel out the *h*'s.