

Use the blank pages provided to show all your work and final answers. Circle Final Answers.

Leave room before, after, and within your work. Your scratch work needs to be included with each problem.

Do not use a separate scratch sheet. I need to see your scratch work and I need to see it next to (within) the problem it supports. That's why I give you blank paper. Write as much as you want, as big as you want.

Do as much of the test as you can without a calculator. If you need a calculator, stop working on that problem and start working on the next problem on a fresh sheet of paper. Come back, later, to do the calculator work.

Most of the points are in the ideas and the techniques. Don't waste time on details if you bog down. **Don't waste time simplifying answers.** It's rarely worth the time for the number of points earned, and sometimes makes things worse! **If you have time at the end, sure, go ahead and pretty things up a little.**

If you submit the problems in the wrong order, that's a **10% deduction from your test score** (One full letter grade).

The next problem goes UNDER the problem before it, not beside it. If not, that's 10% off your test score.

If you're trying to save paper, you're doing it wrong.

1. (5 pts) Use the limit definition of the definite integral to evaluate $\int_{-2}^1 (2x^2 + 5x) dx$. For simplicity, use the limit of the right-endpoint Riemann sum. I'm looking for the correct Riemann Sum.

Bonus (5 pts) Take the limit of your Riemann sum from #1.

2. We find the area of the region bounded by $y = \sqrt{x}$ and $y = x^2$ in two ways.
 - a. (5 pts) Sketch the region.
 - b. (5 pts) Write the area as an integral with respect to x . Draw a representative rectangle on the sketch from part a.
 - c. (5 pts) Evaluate the integral from part b.
 - d. (5 pts) Sketch the region again.
 - e. (5 pts) Write the area as an integral with respect to y . Draw a representative rectangle on the sketch from part d.
 - f. (5 pts) Evaluate the integral from part e.
 - g. (5 pts bonus) Compare your results from parts c and f.

- h. (5 pts) Suppose we rotated the region about the line $x = -2$. Sketch the graph, and write the integral representing the volume of the solid of revolution obtained. Show a representative disc or washer.

Bonus (5 pts) Evaluate the integral in #2h.

3. We explore absolute value. Let $f(x) = 2\cos(x) + 1$

- a. (5 pts) Sketch a complete graph of $f(x)$ on the interval $[0, 2\pi]$.
- b. (5 pts) Evaluate $\int_0^{2\pi} f(x) dx$.
- c. (5 pts) Sketch a complete graph of $g(x) = |2\cos(x) + 1| = |f(x)|$ on the interval $[0, 2\pi]$.
- d. (5 pts) Evaluate $\int_0^{2\pi} g(x) dx$.

4. Evaluate the indefinite integrals. **Do Not Simplify.**

- a. (5 pts) $\int (3x + 5)^{-3} dx$
- b. (5 pts) $\int (3x + 5)^6 x^2 dx$
- c. (5 pts) $\int \csc^6(x) \cot(x) dx$
- d. (5 pts) $\int (2x - 5) \cdot e^{x^2 - 5x} dx$

5. Suppose I'm pacing back and forth, thinking my usual deep thoughts, and my rate of speed is given by $r(t)$, in feet per second. Tell me what the following integrals represent:

- a. (5 pts) $\int_0^{3600} |r(t)| dt$
- b. (5 pts) $\int_0^{3600} r(t) dt$

6. Perform the indicated differentiation. **Do not simplify:**

- a. (5 pts) $\frac{d}{dx} \int_0^x \frac{\sin(5t)}{\sqrt{10t^2 + 4}} dt$
- b. (5 pts) $\frac{d}{dx} \int_{x^2}^{\sqrt{x}} \frac{\sin(5t)}{\sqrt{10t^2 + 4}} dt$

7. The function $f(x) = x^2 - 4x + 9$ is 1-to-1 on the restricted domain $D = [2, \infty)$.

- a. (5 pts) Find the inverse function $f^{-1}(x)$. State its domain and range.
- b. (5 pts) Find $(f^{-1})'(21)$, directly, by differentiating your answer for part a.
- c. (5 pts) Find $(f^{-1})'(21)$ by applying a theorem regarding derivatives of inverse functions.
8. (5 pts each) Find the derivative with respect to x . **Do not simplify.**
- a. (5 pts) $y = 14 \cdot 2^{\sqrt{x}}$
- b. (5 pts) $y = \ln\left(\frac{x^{2/3}\sqrt{2x+1}}{\sin^4(x)}\right)$
- c. (5 pts) $y = \log_7(x^3 + 2x)$
- d. (5 pts) $y = [x^2 + 5x]^{\tan(x)}$
9. (5 pts) The half-life of Carbon-14 is approximately 5730 years. How old is the ash from a Stone-Age firepit if there is 8% of its natural radioactive C-14 remaining?