4:30 pm - 5:00 pm **5:30 pm - 6:09**

1 $\int_{-2}^{1} (2x^2+5x)\,dx$

$\Delta x = \frac{b-a}{n} = \frac{1-(-2)}{n} = \frac{3}{n}$

$x_k = a + k\Delta x = -2 + k\left(\frac{3}{n}\right) = -2 + \frac{3k}{n}$

$x_k^2 = \left(-2+\frac{3k}{n}\right)^2 = \left(\frac{3k}{n}-2\right)^2 = \left(\frac{3k}{n}\right)^2 - 2\left(\frac{3k}{n}\right)(2) + 2^2$

$= \frac{9k^2}{n^2} - \frac{12k}{n} + 4$

Area $\approx \Delta x \sum_{k=1}^{n} f(x_k) = \frac{3}{n}\sum_{k=1}^{n}\left(2x_k^2 + 5x_k\right)$

$= \frac{3}{n}\sum_{k=1}^{n}\left[2\left(\frac{9k^2}{n^2} - \frac{12k}{n} + 4\right) + 5\left(-2+\frac{3k}{n}\right)\right]$

$= \frac{3}{n}\sum_{k=1}^{n}\left[\frac{18k^2}{n^2} - \frac{24k}{n} + 8 - 10 + \frac{15k}{n}\right]$

$= \frac{3}{n}\sum_{k=1}^{n}\left[\frac{18}{n^2}k^2 - \frac{9k}{n} - 2\right] = \frac{3}{n}\cdot\frac{18}{n^2}\sum_{k=1}^{n} k^2 - \frac{3}{n}\cdot\frac{9}{n}\sum_{k=1}^{n} k - \frac{3}{n}\sum_{k=1}^{n} 2$

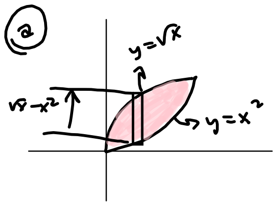
$= \frac{54}{n^3}\left(\frac{n^3 + \cdots}{3}\right) - \frac{27}{n^2}\left(\frac{n^2+\cdots}{2}\right) - \frac{3}{n}\cdot 2n$

$\xrightarrow{n\to\infty} \frac{54}{3} - \frac{27}{2} - 6 = 18 - 6 - \frac{27}{2} = \frac{12}{1}\cdot\frac{2}{2} - \frac{27}{2} = \frac{24-27}{2} = -\frac{3}{2}$

(2) $y=\sqrt{x}$, $y=x^2$

$x=y^2$, $x=\sqrt{y}$

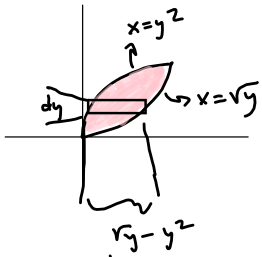
(a)



(b) $\text{Area} = \int_0^1 (\sqrt{x}-x^2)\,dx$

(c) $= \int_0^1 (x^{\frac{1}{2}}-x^2)\,dx = \left[\frac{2}{3}x^{\frac{3}{2}} - \frac{x^3}{3}\right]_0^1 = \frac{2}{3}-\frac{1}{3} = \boxed{\frac{1}{3} = \text{Area}}$

(d)



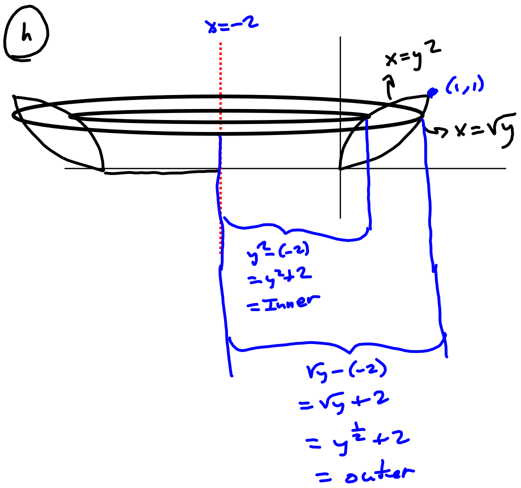
Area

(e) $\text{Area} = \int_0^1 (\sqrt{y}-y^2)\,dy = \int_0^1 (y^{\frac{1}{2}}-y^2)\,dy = \left[\frac{2}{3}y^{\frac{3}{2}} - \frac{y^3}{3}\right]_0^1 = \frac{2}{3}-\frac{1}{3} = \boxed{\frac{1}{3}}$

(f) $\boxed{= \text{Area}}$

(g) They're the same.

(h)



Area

$\text{Volume} = \pi\int_0^1 (\text{outer}^2 - \text{Inner}^2)\,dy = \pi\int_0^1 \left((y^{\frac{1}{2}}+2)^2 - (y^2+2)^2\right)dy$

BONUS

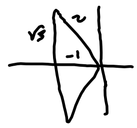
$\text{volume} = \pi\int_0^1 \left(y+4y^{\frac{1}{2}}+4 - (y^4+4y^2+4)\right)dy$

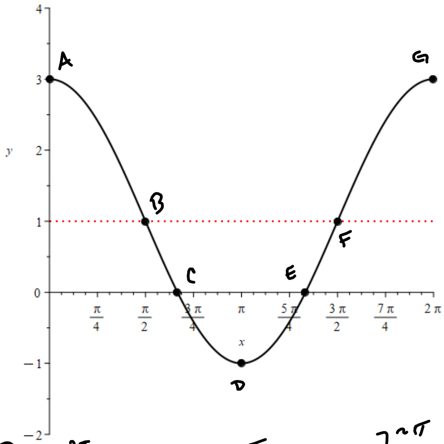
$= \pi\int_0^1 (y+4y^{\frac{1}{2}} - y^4 - 4y^2)\,dy = \left[\frac{y^2}{2} + 4\left(\frac{2}{3}y^{\frac{3}{2}}\right) - \frac{y^5}{5} - \frac{4}{3}y^3\right]_0^1 \pi$

$= \pi\left[\frac{1}{2}+\frac{8}{3}-\frac{1}{5}-\frac{4}{3}\right] = \left[\frac{1}{2}\cdot\frac{15}{15} + \frac{8}{3}\cdot\frac{10}{10} - \frac{1}{5}\cdot\frac{6}{6} - \frac{4}{3}\cdot\frac{10}{10}\right]\pi$

$\frac{15+80-6-40}{30}\pi = \pi\left(\frac{95-46}{30}\right) = \boxed{\frac{49}{30}\pi = \text{Volume}}$

(3) $f(x) = 2\cos(x) + 1 \overset{\text{set}}{=} 0 \longrightarrow \cos(x) = -\frac{1}{2} \Longrightarrow x = \frac{2\pi}{3}, \frac{4\pi}{3}$

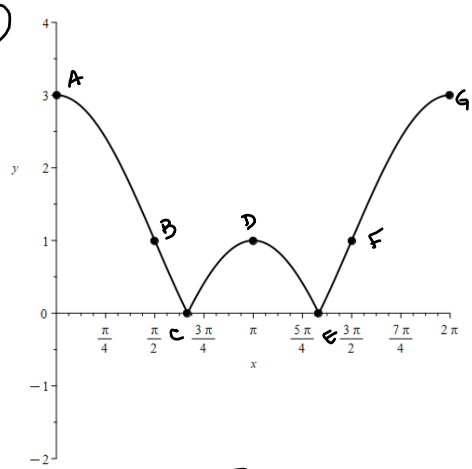




$A = (0,3)$ MAX
$B = (\frac{\pi}{2}, 1)$ IP
$C = (\frac{2\pi}{3}, 0)$ x-int
$D = (\pi, -1)$ MIN
$E = (\frac{4\pi}{3}, 0)$ x-int
$F = (\frac{3\pi}{2}, 1)$ IP
$G = (2\pi, 3)$ MAX

(b) $\int_0^{2\pi}(2\cos(x)+1)\,dx = \left[2\sin(x)+x\right]_0^{2\pi} = 0 + 2\pi - (0+0) = 2\pi$

(c)



$A = (0,3)$ MAX
$B = (\frac{\pi}{2}, 1)$ IP
$C = (\frac{2\pi}{3}, 0)$ x-int
$D = (\pi, 1)$ MAX
$E = (\frac{4\pi}{3}, 0)$ X-INT
$F = (\frac{3\pi}{2}, 1)$ IP
$G = (2\pi, 3)$ MAX

(d) $\int_0^{2\pi} |2\cos(x)+1|\,dx \overset{\boxed{M1}}{=} \int_0^{\frac{2\pi}{3}}(2\cos(x)+1)\,dx - \int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}}(2\cos(x)+1)\,dx + \int_{\frac{4\pi}{3}}^{2\pi}(2\cos(x)+1)\,dx \overset{\boxed{M2}}{=} 2\left[\int_0^{\frac{2\pi}{3}}(2\cos(x)+1)\,dx - \int_{\frac{2\pi}{3}}^{\pi}(2\cos(x)+1)\,dx\right]$ (By symmetry)

$= \boxed{M1}\ \left[2\sin(x)+x\right]_0^{\frac{2\pi}{3}} - \left[2\sin(x)+x\right]_{\frac{2\pi}{3}}^{\frac{4\pi}{3}} + \left[2\sin(x)+x\right]_{\frac{4\pi}{3}}^{2\pi}$

$= 2\left(\frac{\sqrt{3}}{2}\right) + \frac{2\pi}{3} - (0) - \left[2\left(-\frac{\sqrt{3}}{2}\right) + \frac{4\pi}{3} - \left(2\left(\frac{\sqrt{3}}{2}\right) + \frac{2\pi}{3}\right)\right] + \left(2\pi - \left(2\left(-\frac{\sqrt{3}}{2}\right) + \frac{4\pi}{3}\right)\right)$

$= \sqrt{3} + \frac{2\pi}{3} + \sqrt{3} - \frac{4\pi}{3} + \sqrt{3} + \frac{2\pi}{3} + 2\pi - \left(-\sqrt{3} - \frac{4\pi}{3}\right)$

$= \boxed{4\sqrt{3} + \frac{2\pi}{3}}$

$\boxed{M2}\ 2\int_0^{\frac{2\pi}{3}}(2\cos(x)+1)\,dx - 2\int_{\frac{2\pi}{3}}^{\pi}(2\cos(x)+1)\,dx$

$= 2\left[\left(2\sin(x)+x\right)_0^{\frac{2\pi}{3}} - \left[2\sin(x)+x\right]_{\frac{2\pi}{3}}^{\pi}\right]$

$= 2\left[2\left(\frac{\sqrt{3}}{2}\right) + \frac{2\pi}{3} - \left[\pi - 2\left(\frac{\sqrt{3}}{2}\right) - \frac{2\pi}{3}\right]\right]$

$= 2\left[\sqrt{3} + \frac{2\pi}{3} - \pi + \sqrt{3} + \frac{2\pi}{3}\right] = 2\left[2\sqrt{3} + \frac{\pi}{3}\right] = 4\sqrt{3} + \frac{2\pi}{3}$ ✓

4 a $\int (3x+5)^{-3}dx = \int u^{-3}\frac{du}{3} = \frac{1}{3}\int u^{-3}du = \frac{1}{3}\left(-\frac{1}{2}\right)u^{-2}+C$

$= \boxed{-\frac{1}{6}(3x+5)^{-2}+C}$

$u = 3x+5$

$du = 3dx$

$\frac{du}{3} = dx$

b $\int (3x+5)^6 x^2\,dx = \frac{1}{9}\int u^6(u^2-10u+25)\cdot\frac{du}{3}$

$u = 3x+5$

$du = 3dx$

$dx = \frac{du}{3}$

$u-5 = 3x$

$x = \frac{u-5}{3}$

$x^2 = \left(\frac{u-5}{3}\right)^2 = \frac{u^2-10u+25}{9}$

$= \frac{1}{27}\int (u^8 - 10u^7 + 25u)\,du = \frac{1}{27}\left(\frac{u^9}{9} - \frac{10}{8}u^8 + \frac{25}{7}u^7\right)+C$

$= \boxed{\frac{1}{27}\left(\frac{(3x+5)^9}{9} - \frac{5}{4}(3x+5)^8 + \frac{25}{7}(3x+5)^7\right)+C}$

c $\int \csc^6(x)\cot(x)\,dx = -\int \csc^5(x)\left(-\csc(x)\cot(x)\right)dx$

$= -\int u^5 du = -\frac{u^6}{6} + C = \boxed{-\frac{\csc^6(x)}{6}+C}$

d $\int (2x-5)e^{x^2-5x}dx = \int e^u(2x-5)\frac{du}{2x-5} = \int e^u du = e^u + C$

$= \boxed{e^{x^2-5x}+C}$

$u = x^2-5x$

$du = (2x-5)dx$

$dx = \frac{du}{2x-5}$

5 $r(t)$ = rate of speed in ft per second, as a function of t = time in seconds.

a $\int_0^{3600} |r(t)|\,dt$ = Total distance traveled, in feet, in 1 hour

b $= \int_0^{3600} r(t)\,dt$ = Net change, in feet, after 1 hour.

6 a $\frac{d}{dx}\int_0^x \frac{\sin(5t)}{\sqrt{10t^2+4}}\,dt = \boxed{\frac{\sin(5x)}{\sqrt{10x^2+4}}}$

b $\frac{d}{dx}\int_{x^2}^{\sqrt{x}} \frac{\sin(5t)}{\sqrt{10t^2+4}}\,dt$

$= \frac{d}{dx}\left[\int_{x^2}^{0} \sim + \int_0^{\sqrt{x}} \sim\right] = \frac{d}{dx}\left[-\int_0^{x^2}\frac{\sin(5t)}{\sqrt{10t^2+4}}\,dt + \int_0^{\sqrt{x}}\frac{\sin(5t)}{\sqrt{10t^2+4}}\,dt\right]$

$= \left(-\frac{\sin(5x^2)}{\sqrt{10(x^2)^2+4}}\right)\left(2x\right) + \left(\frac{\sin(5\sqrt{x})}{\sqrt{10(\sqrt{x})^2+4}}\right)\left(\frac{1}{2}x^{-\frac{1}{2}}\right)$

(7) $f(x) = x^2 - 4x + 9$ on $[2, \infty)$

METHOD 1
complete the square

$f(x) = x^2 - 4x + 2^2 - 4 + 9$

$= (x-2)^2 + 5 = y \longrightarrow$

$(x-2)^2 = y - 5 \longrightarrow$

$x - 2 = \pm\sqrt{y-5} \longrightarrow$

$x = 2 \pm \sqrt{y-5}$

want "+"

$\boxed{f^{-1}(x) = 2 + \sqrt{x-5}}$

METHOD 2
Quadratic Formula

$x^2 - 4x + 9 = y \longrightarrow$

$x^2 - 4x + (9 - y) = 0$

$a = 1,\ b = -4,\ c = 9 - y \longrightarrow$

$b^2 - 4ac = 16 - 4(1)(9 - y)$

$= 16 - 36 + 4y$

$= 4y - 20 = 4(y-5)$

$\Longrightarrow x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{4 \pm \sqrt{4(y-5)}}{2}$

$= \dfrac{4 \pm 2\sqrt{y-5}}{2} = 2 \pm \sqrt{y-5}$

Take the "+":

$\boxed{f^{-1}(x) = 2 + \sqrt{x-5}}$

(b) $(f^{-1})'(21)$:

$f^{-1}(x) = 2 + (x-5)^{\frac{1}{2}}$

$\Longrightarrow (f^{-1})'(x) = \frac{1}{2}(x-5)^{-\frac{1}{2}} = \dfrac{1}{2\sqrt{x-5}} \longrightarrow$

$(f^{-1})'(21) = \dfrac{1}{2\sqrt{21-5}} = \dfrac{1}{2(\sqrt{16})} = \dfrac{1}{2(4)} = \boxed{\dfrac{1}{8} = (f^{-1})'(21)}$

(c) $(f^{-1})'(21) = \dfrac{1}{f'(f^{-1}(21))}$

$f'(x) = 2x - 4$

FIND $f^{-1}(21)$ by solving

$f(x) = 21$

$\Longrightarrow x^2 - 4x + 9 = 21$

$x^2 - 4x - 12 = 0$

$(x-6)(x+2) = 0$

$x \in \{-2, 6\}$,

$x = -2 \notin [2, \infty)$

$\boxed{x = 6} = f^{-1}(21)$

Put it together:

$\dfrac{1}{f'(f^{-1}(21))} = \dfrac{1}{f'(6)} = \dfrac{1}{2(6)-4} = \dfrac{1}{12-4} = \boxed{\dfrac{1}{8} = (f^{-1})'(21)}$

8 a $y = 14 \cdot 2^{\sqrt{x}} = 14 \cdot 2^{x^{\frac{1}{2}}} \longrightarrow$

$y' = \boxed{14 \ln(2)(2^{\sqrt{x}})(\tfrac{1}{2}x^{-\frac{1}{2}})}$

b $y = \ln\left(\dfrac{x^{\frac{2}{3}}\sqrt{2x+1}}{\sin^4(x)}\right) = \frac{2}{3}\ln(x) + \frac{1}{2}\ln(2x+1) - 4\ln(\sin(x))$

$\longrightarrow \boxed{y' = \frac{2}{3}\left(\frac{1}{x}\right) + \frac{1}{2}\left(\frac{2}{2x+1}\right) - 4\left(\frac{\cos(x)}{\sin(x)}\right)}$

c $y = \log_7(x^3 + 2x) \longrightarrow$

$\boxed{y' = \frac{1}{\ln(7)}\left(\frac{3x^2+2}{x^3+2x}\right)}$

d $y = (x^2+5x)^{\tan(x)} \longrightarrow$

$\ln(y) = \tan(x)\ln(x^2+5x) \longrightarrow$

$\frac{y'}{y} = \sec^2(x)\ln(x^2+5x) + (\tan(x))\left(\frac{2x+5}{x^2+5x}\right)$

$\longrightarrow \boxed{y' = \left[\sec^2(x)\ln(x^2+5x) + \frac{2x+5}{x^2+5x}\tan(x)\right]\left((x^2+5x)^{\tan(x)}\right)}$

9. $\frac{1}{2}$ life is 5730 yrs. 8% of original C_{14} remains.

$N(t) = N_0 e^{kt}$, where $N_0 = N(0)$ = initial amt of C_{14}

t = time in years, and k is decay constant.

$$N(5730) = \frac{1}{2} N_0$$

$$N_0 e^{5730k} = \frac{1}{2} N_0$$

$$e^{5730k} = \frac{1}{2}$$

$$5730k = \ln(1/2) = -\ln(2)$$

$$\boxed{k = \frac{-\ln(2)}{5730}}$$

Find t for $N(t) = 0.08$:

$$N_0 e^{kt} = .08 N_0 \Longrightarrow$$

$$e^{kt} = .08 \Longrightarrow$$

$$kt = \ln(.08) \Longrightarrow$$

$$t = \frac{\ln(.08)}{k} = \frac{\ln(.08)}{\frac{-\ln(2)}{5730}} = \boxed{\frac{-5730\ln(.08)}{\ln(2)} = t}$$

$$\approx \boxed{20879.29597 \text{ yrs}}$$