MAT 2410-R91 Written Midterm, Fall, 2024 (5 pts) Name ________________

120 Points Covers Chapters 1 and 2

Please print your name in the space provided, above-right.

Do all your work and circle all your final answers on the blank paper provided by the test proctors. The only exception to this rule is the graph of f' for #8.

You don't need to write out the questions, but you do need to write out your answers as completely as possible.

Don't spend more than 2 or 3 minutes on a problem before moving on to the next. Get something written down, leave lots of room, and come back to it after you've made your first pass at the whole test. When in doubt, start a fresh sheet of paper for the next problem.

1. (5 pts each) Evaluate the following limits, if they exist. If one does not exist, explain why.

   a. $\lim_{x\to 2^-} \frac{x^2+5x-14}{|x-2|}$ b. $\lim_{x\to 2^+} \frac{x^2+5x-14}{|x-2|}$ c. $\lim_{x\to 2} \frac{x^2+5x-14}{|x-2|}$

2. Consider the piecewise-defined function $f(x) = \begin{cases} x^2-4x+5 & \text{if } x<3 \\ -\frac{1}{2}x+\frac{7}{2} & \text{if } x \geq 3 \end{cases}$.

   a. (5 pts) Sketch the graph of $f(x)$. Label the x- and y-intercepts, the suture point(s), and the vertex of the quadratic piece, if it's in the picture. When I say "Label," I mean an ordered pair, like (0, 5), next to the point.

   b. (5 pts) On what interval(s) is $f(x)$ continuous? Explain.

3. (5 pts) Simplify $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ for $f(x)=x^2-4x+5$.

4. The point $P(2,-4)$ lies on the graph of $f(x)=x^2-2x-4$.

   a. (5 pts) Write the equation of the tangent line to $f(x)$ at P.

   b. (5 pts) Sketch a graph of $f(x)$ and the tangent line to $f(x)$ at the point P.

5. (5 pts) Sketch a plausible graph of a mostly-smooth function f that has the following properties. (Note: That very last condition is a later topic, "limits *at* infinity." So, 5 bonus points for the horizontal asymptote.)

   c. $\lim_{x\to -3^-} f(x)=7$ f. $\lim_{x\to 2^-} f(x)=\infty$ i. $\lim_{x\to -\infty} f(x)=2$

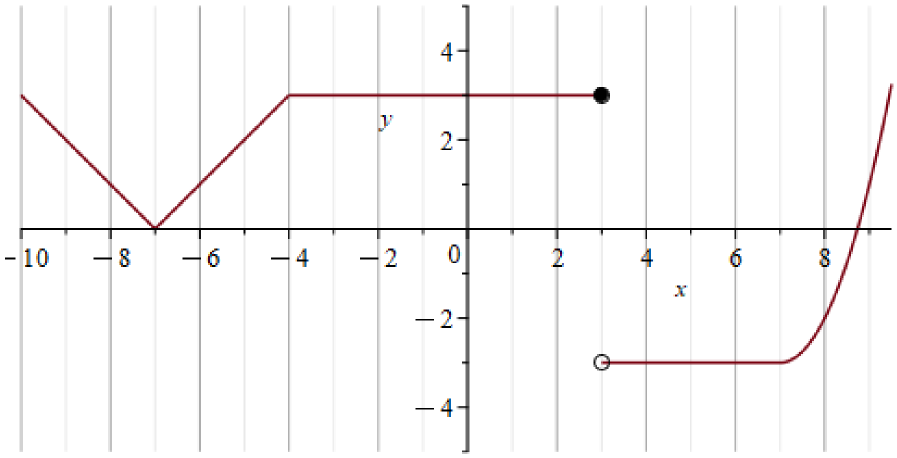
   d. $\lim_{x\to -3^+} f(x)=-4$ g. $\lim_{x\to 2^+} f(x)=\infty$ j. $\lim_{x\to \infty} f(x)=1$

   e. $f(-3)=7$ h. $f(2)=3$

6. (5 pts) Prove that $\lim_{x\to 2}(3x-7)=-1$, using the $\varepsilon-\delta$ definition of limit.(5 pts)

7. (5 pts) Prove that the equation $f(x)=x^2-4x\sin(x)+2$ has a root in the interval $(0,2)$, but *do not solve*!

8. (5 pts) The graph of a function f is given below. On the same set of axes, sketch a graph of f' This is the one problem I *want* you to write on this test sheet. Do the rest of your work for all other exercises on the blank paper provided by the proctors.



9. Differentiate the following with respect to the indicated independent variable. **Do not simplify!**

   a. (5 pts) $f(x) = \sqrt[7]{x^6} - 3x^3 + 5\sqrt{x} - \dfrac{7}{x^2}$; x.

   b. (5 pts) $g(x) = \sin(5x)\tan(3x)$; x.

   c. (5 pts) $h(\rho) = \dfrac{\cos(\rho)}{\left(7\rho^2 - 5\rho^{2/3}\right)}$; ρ.

   d. (5 pts) $r(w) = \left(w^2 + 11w + 5\right)^4 (2w+6)^3$; w.

10. Consider the relation $x^2 - 3xy + 4y^2 = \cos(y)$.

   a. (5 pts) Use implicit differentiation to find $y' = \dfrac{dy}{dx}$

   b. (5 pts) Find an equation of the tangent line to the curve at the point $(1,0)$.

11. (5 pts) A woman who is 5 feet tall is walking away from a street light at 3 feet per second. If the light is 18 feet off the ground, how fast is the tip of the woman's shadow moving away from the light when the woman is 10 feet away from the light pole? Round your final answer to 3 digits to the right of the decimal.

12. A man wants to paint the outside of a cube with a coat of paint that is 0.005 inch thick. The sides of the cube are 10 feet by 10 feet. Use a differential to approximate the volume of paint required....

   a. (5 pts) ... in cubic feet. Round your final answer to two decimal places.

   b. (5 pts) ... in gallons. Use 1 cubic foot = 7.48052 gallons (approximately). Round your answer to two decimal places.

BONUS SECTION: Work any 2 bonus questions for up to 10 bonus points.

1. (5 pts) Prove that $\lim_{x\to 2}\left(2x^2 - 3x + 1\right) = 3$, using the $\varepsilon - \delta$ definition of limit.

2. (5 pts) Evaluate $\lim_{h\to 0}\frac{\sqrt{2x+2h}-\sqrt{2x}}{h}$, if it exists. If it does not, state why.

3. (5 pts) See if you can *squeeze* out a *convincing* argument to support the statement

$$f(x)=\begin{cases} x^2\sin\left(\frac{\pi}{x}\right) & \text{if } x\neq 0 \\ 0 & \text{if } x=0 \end{cases}$$ is continuous on $(-\infty,\infty)$.