WP#3

1 5pts Evaluate $\int_{-1}^{2}(x^2-3x)dx$ by the limit defn:

$[a,b]=[-1,2] \Rightarrow$

$\Delta x = \frac{b-a}{n} = \frac{2-(-1)}{n} = \frac{3}{n}$

$x_k = a+k(\Delta x) = -1+k\left(\frac{3}{n}\right) = -1+\frac{3k}{n}$

$f(x_k) = x_k^2 - 3x_k = \left(-1+\frac{3k}{n}\right)^2 - 3\left(-1+\frac{3k}{n}\right)$

$= 1 - 2\left(\frac{3k}{n}\right) + \left(\frac{3k}{n}\right)^2 + 3 - \frac{9k}{n}$

$= 1 - \frac{6k}{n} + \frac{9k^2}{n^2} + 3 - \frac{9k}{n} \longrightarrow$

$= 4 - \frac{15k}{n} + \frac{9}{n^2}k^2 \Longrightarrow$

$\int_{-1}^{2} f(x)dx \approx \Delta x \sum_{k=1}^{n} f(x_k) = \frac{3}{n}\sum_{k=1}^{n}\left(4 - \frac{15}{n}k + \frac{9}{n^2}k^2\right)$

$= \frac{3}{n}\sum 4 - \frac{3}{n}\cdot\frac{15}{n}\sum_{k=1}^{n} k + \frac{3}{n}\cdot\frac{9}{n^2}\sum_{k=1}^{n} k^2$

$= \boxed{\frac{3}{n}\cdot 4n - \frac{45}{n^2}\left[\frac{n^2+\sim}{2}\right] + \frac{27}{n^3}\left[\frac{n^3+\sim}{3}\right]}$

$\xrightarrow{n\to\infty} 12 - \frac{45}{2} + 9 = 21 - \frac{45}{2} = \frac{42}{2} - \frac{45}{2} = \boxed{-\frac{3}{2}}$

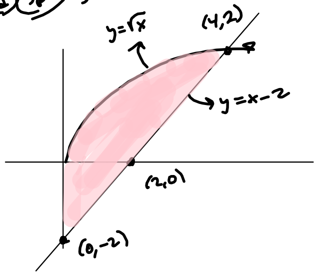
Check w/ FTC II!

$\int_{-1}^{2}(x^2-3x)dx = \left[\frac{x^3}{3} - \frac{3x^2}{2}\right]_{-1}^{2} = \frac{2^3}{3} - \frac{3(2)^2}{2} - \left[\frac{(-1)^3}{3} - \frac{3(-1)^2}{2}\right]$

$= \frac{8}{3} - \frac{12}{2} + \frac{1}{3} + \frac{3}{2} = \frac{9}{3} - \frac{9}{2} = \frac{18-27}{6} = -\frac{9}{6} = \boxed{-\frac{3}{2}}$ ✓

② Area bdd by $y=\sqrt{x}$, $y=x-2$, $x=0$

(a) (5pts) sketch



$\sqrt{x} = x-2$

$x=(x-2)^2 = x^2-4x+4$

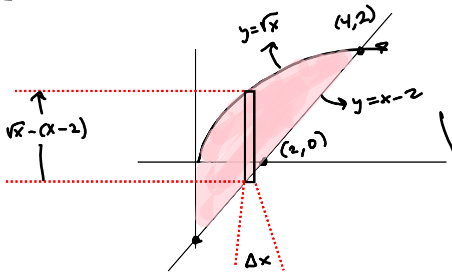
$\Rightarrow x^2-5x+4=0 \Rightarrow$

$(x-1)(x-4)=0 \Rightarrow$

$x\in\{1,4\}$. $x=1$ is extraneous

So $x=4$, $y=\sqrt{4}=2 \rightarrow (4,2)$

(b) Show representative rectangle & write the integral for the area.



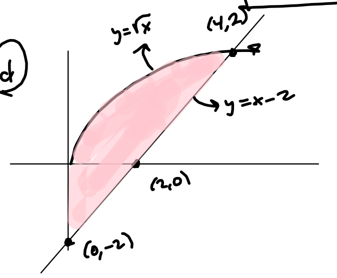
$$\text{Area} = \int_0^4 (\text{upper}-\text{lower})\,dx$$

$$\boxed{= \int_0^4 (\sqrt{x}-(x-2))\,dx}$$

(c) $= \int_0^4 (x^{\frac{1}{2}}-x+2)\,dx = \left[\frac{2}{3}x^{\frac{3}{2}} - \frac{x^2}{2} + 2x\right]_0^4 = \left[\frac{2}{3}(4)^{\frac{3}{2}} - \frac{4^2}{2} + 2(4) - 0\right]$

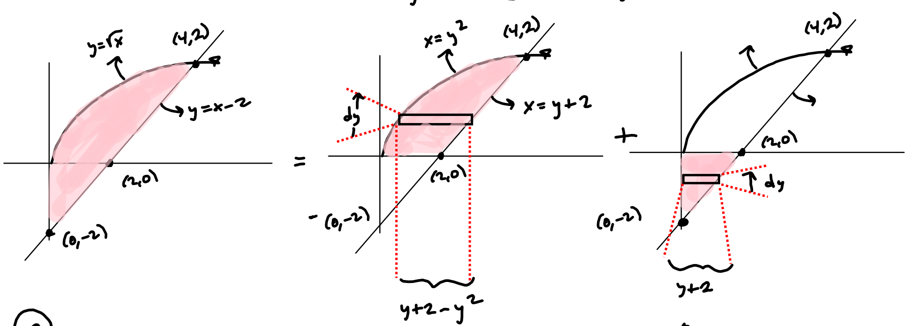
$= \frac{2}{3}(8) - 8 + 8 = \boxed{\frac{16}{3} = \text{Area}}$

(d)



(e) Write integral (actually 2 integrals) and show (a) representative rectangle(s).

Different "left" function for $y\in[-2,0]$ and $y\in[0,2]$:



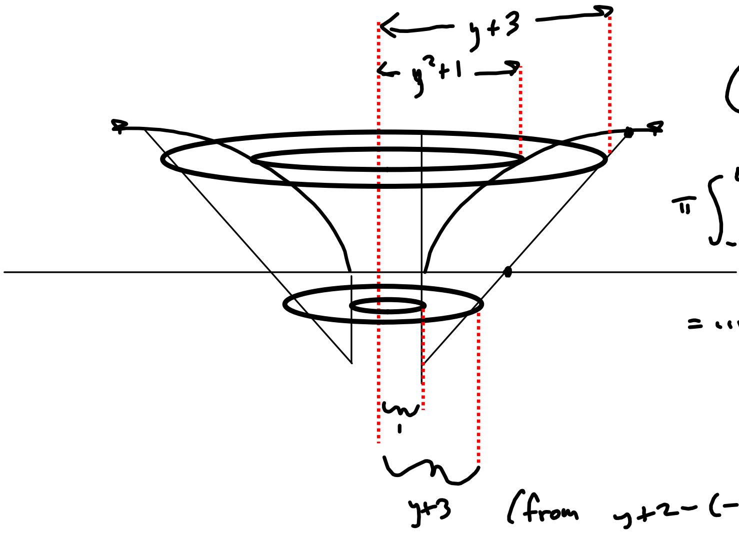
(e) $\int_0^2 (y+2-y^2)\,dy + \int_{-2}^0 (y+2)\,dy$

(f) $= \left[\frac{y^2}{2}+2y-\frac{y^3}{3}\right]_0^2 + \left[\frac{y^2}{2}+2y\right]_{-2}^0$

$= \left(\frac{2^2}{2}+2(2)-\frac{2^3}{3}\right) - (0) + (0) - \left(\frac{(-2)^2}{2}+2(-2)\right)$

$= 2+4-\frac{8}{3} - \left(\frac{4}{2}-4\right) = 6-\frac{8}{3}-(2-4) = 6-\frac{8}{3}+2 = 8-\frac{8}{3} = \frac{24-8}{3} = \boxed{\frac{16}{3}}$

(g) $\frac{16}{3}$ for both!



(h)

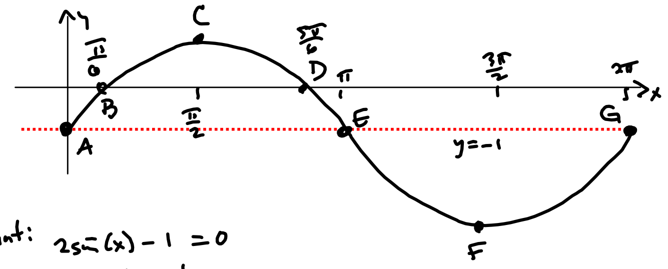
Volume =

$$\pi \int_{-2}^{0} \left(\left((y+3)^2 - 1^2 \right) + \left((y+3)^2 - (y^2+1)^2 \right) \right) dy$$

$$= \ldots = \frac{128\pi}{5}$$

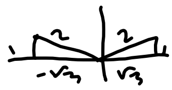
③ $f(x) = 2\sin(x) - 1$

ⓐ Sketch on $[0, 2\pi]$



$A = (0,-1)$ IP
$B = (\frac{\pi}{6}, 0)$ x-int
$C = (\frac{\pi}{2}, 1)$ MAX
$D = (\frac{5\pi}{6}, 0)$ x-int
$E = (\pi, -1)$ IP
$F = (\frac{3\pi}{2}, -3)$ MIN
$G = (2\pi, -1)$ IP

x-int: $2\sin(x) - 1 = 0$
$\sin(x) = \frac{1}{2}$



$x = \frac{\pi}{6}, \frac{5\pi}{6}$

ⓑ $\int_0^{2\pi} f(x)\,dx = \int_0^{2\pi} (2\sin(x) - 1)\,dx = \left[-2\cos(x) - x\right]_0^{2\pi} = -2\cos(2\pi) - 2\pi - (-2\cos(0) - 0)$

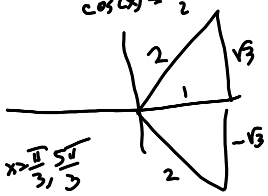
$= -2 - 2\pi + 2 + 0 = -2\pi$

2 versions, b/c #3c is poorly posed

ⓒ Version 1

Graph $y = 2\cos(x) - 1$

$2\cos(x) = 1$
$\cos(x) = \frac{1}{2}$



$x = \frac{\pi}{3}, \frac{5\pi}{3}$

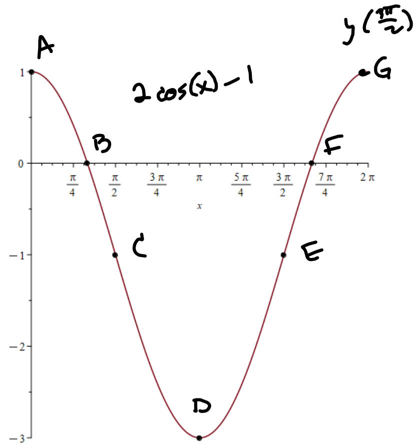
$y(\frac{\pi}{3}) = 2\cos(\frac{\pi}{3}) - 1 = 2(\frac{1}{2}) - 1 = 0$ ✓
$y(\frac{5\pi}{3}) = 2(\cos(\frac{5\pi}{3})) - 1 = 2(\frac{1}{2}) - 1 = 0$ ✓

$y' = -2\sin(x) = 0 \Rightarrow$
$x = 0, \pi, 2\pi$

$y(0) = 2 - 1 = 1$ MAX
$y(\pi) = 2(\cos(\pi)) - 1 = -2 - 1 = -3$ MIN
$y(2\pi) = 2\cos(2\pi) - 1 = +2 - 1 = 1$ MAX

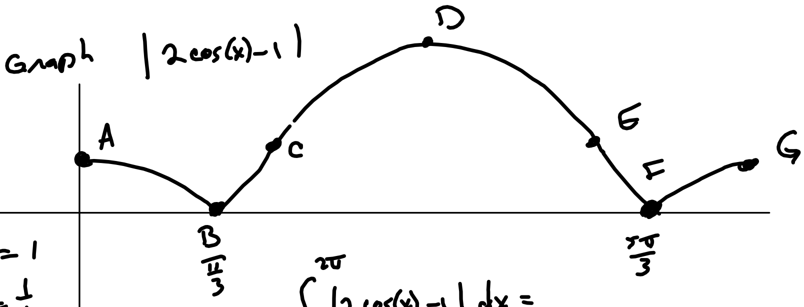
$y'' = -2\cos(x) = 0 \Rightarrow$
$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$y(\frac{\pi}{2}) = -2\cos(\frac{\pi}{2}) - 1 = -1$
$y(\frac{3\pi}{2}) = -2\cos(\frac{3\pi}{2}) - 1 = -1$ $\Big\}$ IP



$A = (0,1)$ MAX
$B = (\frac{\pi}{3}, 0)$ x-int
$C = (\frac{\pi}{2}, -1)$ IP
$D = (\pi, -3)$ MIN
$E = (\frac{3\pi}{2}, -1)$ IP
$F = (\frac{5\pi}{3}, 0)$ x-int
$G = (2\pi, 1)$ MAX

Very few students will do this graph, unfortunately



$A = (0,1)$ Y-INT & IP
$B = (\frac{\pi}{3}, 0)$ X-INT
$C = (\frac{\pi}{2}, 1)$ IP
$D = (\pi, 3)$ MAX
$E = (\frac{3\pi}{2}, 1)$ IP
$F = (\frac{5\pi}{3}, 0)$ X-INT
$G = (2\pi, 1)$ IP

$2\cos(x) = 1$
$\cos(x) = \frac{1}{2}$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

$$\int_0^{2\pi} |2\cos(x) - 1| \, dx =$$

$$= \int_0^{\frac{\pi}{3}} (2\cos(x) - 1)\,dx - \int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} (2\cos(x) - 1)\,dx + \int_{\frac{5\pi}{3}}^{2\pi} (2\cos(x) - 1)\,dx$$

$$= \Big[2\sin(x) - x\Big]_0^{\frac{\pi}{3}} - \Big[2\sin(x) - x\Big]_{\frac{\pi}{3}}^{\frac{5\pi}{3}} + \Big[2\sin(x) - x\Big]_{\frac{5\pi}{3}}^{2\pi}$$

$$= 2\sin(\tfrac{\pi}{3}) - \tfrac{\pi}{3} - (2\sin(0) - 0) - \Big[2\sin(\tfrac{5\pi}{3}) - \tfrac{5\pi}{3} - (2\sin(\tfrac{\pi}{3}) - \tfrac{\pi}{3})\Big] + 2\sin(2\pi) - 2\pi - \left(2\sin(\tfrac{5\pi}{3}) - \tfrac{5\pi}{3}\right)$$

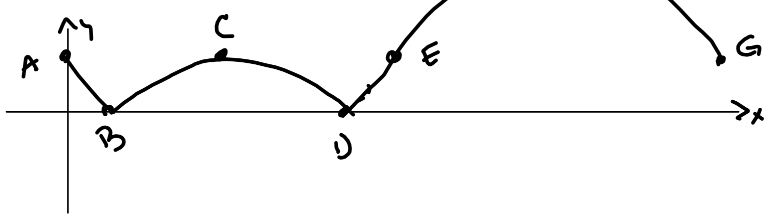
$$= 2\left(\tfrac{\sqrt{3}}{2}\right) - \tfrac{\pi}{3} - \Big[2\left(-\tfrac{\sqrt{3}}{2}\right) - \tfrac{5\pi}{3} - \left(2\left(\tfrac{\sqrt{3}}{2}\right) - \tfrac{\pi}{3}\right)\Big] + 0 - 2\pi - 2\left(-\tfrac{\sqrt{3}}{2}\right) + \tfrac{5\pi}{3}$$

$$= \sqrt{3} - \tfrac{\pi}{3} + \sqrt{3} + \tfrac{5\pi}{3} + \sqrt{3} - \tfrac{\pi}{3} - 2\pi + \sqrt{3} + \tfrac{5\pi}{3}$$

$$= 4\sqrt{3} + \tfrac{2\pi}{3}$$

Version 2

Graph $|2\sin(x)-1|$



$A = (0,1)$ IP & x-int

$B = (\frac{\pi}{6},0)$ x-int

$C = (\frac{\pi}{2},1)$ MAX

$D = (\frac{5\pi}{6},0)$ x-int

$E = (\pi,1)$ IP

$F = (\frac{3\pi}{2},3)$ MAX

$G = (2\pi,1)$ IP

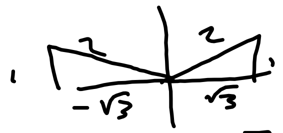
$$\int_0^{2\pi} |2\sin(x)-1|\,dx$$

$$= -\int_0^{\frac{\pi}{6}}(2\sin(x)-1)\,dx + \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}}(2\sin(x)-1)\,dx - \int_{\frac{5\pi}{6}}^{2\pi}(2\sin(x)-1)\,dx$$

$$= -\Big[-2\cos(x)-x\Big]_0^{\frac{\pi}{6}} + \Big[-2\cos(x)-x\Big]_{\frac{\pi}{6}}^{\frac{5\pi}{6}} - \Big[-2\cos(x)-x\Big]_{\frac{5\pi}{6}}^{2\pi}$$

$$= -\Big[-2\cos\left(\tfrac{\pi}{6}\right)-\tfrac{\pi}{6}-\left(-2\cos(0)-0\right)\Big] + \Big[-2\cos\left(\tfrac{5\pi}{6}\right)-\tfrac{5\pi}{6}-\left(-2\cos\left(\tfrac{\pi}{6}\right)-\tfrac{\pi}{6}\right)\Big]$$

$$-\Big[-2\cos(2\pi)-2\pi-\left(-2\cos\left(\tfrac{5\pi}{6}\right)-\tfrac{5\pi}{6}\right)\Big]$$



$\frac{\pi}{6}, \frac{5\pi}{6}$

$$-\Big[-2\left(\tfrac{\sqrt{3}}{2}\right)-\tfrac{\pi}{6}-(-2)\Big] + \Big[-2\left(-\tfrac{\sqrt{3}}{2}\right)-\tfrac{5\pi}{6}-\left(-2\left(\tfrac{\sqrt{3}}{2}\right)-\tfrac{\pi}{6}\right)\Big]$$

$$-\Big[-2-2\pi-\left(-2\left(-\tfrac{\sqrt{3}}{2}\right)-\tfrac{5\pi}{6}\right)\Big]$$

$$= -\Big[\sqrt{3}-\tfrac{\pi}{6}+2\Big] + \Big[\sqrt{3}-\tfrac{5\pi}{6}-\left(-\sqrt{3}-\tfrac{\pi}{6}\right)\Big] - \Big[-2-2\pi-\left(\sqrt{3}-\tfrac{5\pi}{6}\right)\Big]$$

$$= +\sqrt{3}+\tfrac{\pi}{6}-2+\sqrt{3}-\tfrac{5\pi}{6}+\sqrt{3}+\tfrac{\pi}{6}+2+2\pi+\sqrt{3}-\tfrac{5\pi}{6}$$

$$= 4\sqrt{3}+\tfrac{2\pi}{3} \checkmark$$

$$\tfrac{3\pi}{2}-\tfrac{5\pi}{6} = \tfrac{9\pi-5\pi}{6} = \tfrac{4\pi}{6} = \tfrac{2\pi}{3}$$

#4 (a) $\int (2x-3)^4 dx = \frac{1}{2}\underbrace{\int (2x-3)^4 (2dx)}_{\frac{1}{2}\int u^4 du = \frac{1}{2}\left(\frac{u^5}{5}\right)+C} = \boxed{\frac{1}{2}\frac{(2x-3)^5}{5} + C = \frac{(2x-3)^5}{10} + C}$

$u = 2x-3$
$du = 2dx$

(b) $\int x^2(2x-3)^4 dx$

$$\begin{array}{ccccccccc} & & & & 1 & & & & \\ & & & 1 & & 1 & & & \\ & & 1 & & 2 & & 1 & & \\ & 1 & & 3 & & 3 & & 1 & \\ 1 & & 4 & & 6 & & 4 & & 1 \end{array}$$

Method 1: Brute Force

$$\int x^2\left((2x)^4 - 4(2x)^3(3) + 6(2x)^2(3)^2 - 4(2x)(3)^3 + 3^4\right) dx$$

$$= \int x^2\left(16x^4 - 96x^3 + 24(9)x^2 - 216x + 81\right) dx$$

$$= \int \left(16x^6 - 96x^5 + 216x^4 - 216x^3 + 81x^2\right) dx$$

$$\begin{array}{r} 240 \\ -24 \\ \hline 216 \end{array}$$

$$= \frac{16}{7}x^7 - \frac{96}{6}x^6 + \frac{216}{5}x^5 - \frac{216}{4}x^4 + \frac{81}{3}x^3 + C$$

$$= \boxed{\frac{16}{7}x^7 - 16x^6 + \frac{216}{5}x^5 - 54x^4 + 27x^3 + C}$$

Method 2: *u*-substitution:

$\int x^2(2x-3)^4 dx$

$u = 2x-3$

$\Rightarrow du = 2dx$ and $u+3 = 2x$

$\Rightarrow dx = \frac{du}{2}$ $\Rightarrow \frac{u+3}{2} = x$

$\Rightarrow x^2 = \left(\frac{u+3}{2}\right)^2 = \frac{1}{4}(u^2+6u+9)$

This gives $\int x^2(2x-3)^4 dx = \int \frac{1}{4}(u^2+6u+9)(u^4)\frac{du}{2} = \frac{1}{8}\int (u^6+6u^5+9u^4)\, du$

$$= \frac{1}{8}\left[\frac{u^7}{7} + u^6 + \frac{9}{5}u^5 + C\right]$$

$$= \frac{1}{8}\left[\frac{(2x-3)^7}{7} + (2x-3)^6 + \frac{9}{5}(2x-3)^5\right] + \hat{C}$$ is good enough for me!

I don't want to expand that all out, but it turns out to be the same answer.

(c) $\int \sec^4(x)\tan(x)\,dx = \int \sec^3(x)\left(\sec(x)\tan(x)\,dx\right)$

$= \int u^3\,du = \frac{u^4}{4} + C = \boxed{\frac{\sec^4(x)}{4} + C}$

Let $u = \sec(x)$.
Then $du = \sec(x)\tan(x)\,dx$
Nice!

(d) $\int \sin(x)e^{\cos(x)}dx$.

Let $u = \cos(x)$ then
$du = -\sin(x)\,dx \longrightarrow$

$\frac{du}{-\sin(x)} = dx \longrightarrow$

$\int \sin(x)e^u \frac{du}{-\sin(x)} = -\int e^u\,du = -e^u + C = \boxed{-e^{\cos(x)} + C}$

5) $r(t)$ = rate of speed in ft/sec.

a) $\int_0^{3600} |r(t)|\,dt = \underbrace{\text{Total distance}}_{\text{in feet}}$ traveled. in 3600 sec = 1 hour

b) $\int_0^{3600} r(t)\,dt = \underbrace{\text{Net Change}}_{\text{in feet}}$ in my position in 1 hour.

6) a) $\frac{d}{dx}\int_0^x \frac{\cos(3t)}{\sin(t)+4}\,dt = \boxed{\frac{\cos(3x)}{\sin(x)+4}}$

b) $\frac{d}{dx}\int_{x^2}^{\cos(x)} \frac{\cos(3t)}{\sin(t)+4}\,dt = \frac{d}{dx}\int_{x^2}^{0} \frac{\cos(3t)}{\sin(t)+4}\,dt + \frac{d}{dx}\int_{0}^{\cos(x)} \frac{\cos(3t)}{\sin(t)+4}\,dt$

$= -\frac{d}{dx}\int_0^{x^2} \frac{\cos(3t)}{\sin(t)+4}\,dt + \frac{d}{dx}\int_0^{\cos(x)} \frac{\cos(3t)}{\sin(t)+4}\,dt$

$= -\left(\frac{\cos(3(x^2))}{\sin(x^2)+4}\right)(2x) + \left(\frac{\cos(3\cos(x))}{\sin(\cos(x))+4}\right)(-\sin(x))$

(7) $y = x^2-5x+11$ on $[\frac{5}{2},\infty)$

(a) $= x^2-5x+\left(\frac{5}{2}\right)^2-\frac{25}{4}+\frac{44}{4}$

$= \left(x-\frac{5}{2}\right)^2+\frac{19}{4} = y$

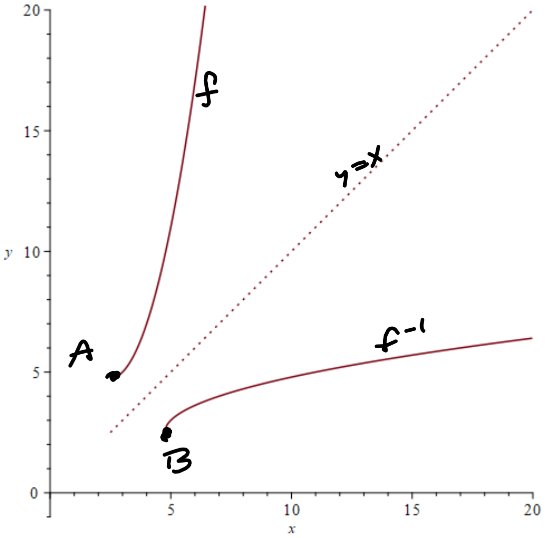
$\Rightarrow \left(x-\frac{5}{2}\right)^2 = y-\frac{19}{4}$

$\Rightarrow x-\frac{5}{2} = \pm\sqrt{y-\frac{19}{4}}$

$\Rightarrow x = \frac{5}{2}\pm\sqrt{y-\frac{19}{4}}$ OR $\frac{5}{2}\pm\sqrt{\frac{4y-19}{4}} = \frac{5\pm\sqrt{4y-19}}{2}$

$\boxed{f^{-1}(x) = \frac{5+\sqrt{4x-19}}{2}}$

$\boxed{\mathcal{D}(f^{-1}) = [\frac{19}{4},\infty) \quad \mathcal{R}(f^{-1}) = [\frac{5}{2},\infty)}$



$A = (\frac{5}{2},\frac{19}{4})$

$B = (\frac{19}{4},\frac{5}{2})$

You can also use the quadratic formula to invert this quadratic function:

$x^2-5x+11 = y$

$x^2-5x+(11-y) = 0$

$a=1, b=-5, c=11-y$

$b^2-4ac = 25-4(11-y) = 25-44+4y$

$= 4y-19$

$x = \frac{-b+\sqrt{b^2-4ac}}{2a}$

$= \frac{5\pm\sqrt{4y-19}}{2}$ Take the top $\pm$: $x = \frac{5+\sqrt{4y-19}}{2}$

$f^{-1}(x) = \frac{5+\sqrt{4x-19}}{2}$. See?

(b) $f^{-1}(x) = \frac{5+\sqrt{4x-19}}{2} = \frac{5}{2} + \frac{1}{2}(4x-19)^{\frac{1}{2}}$

$\Rightarrow (f^{-1})'(x) = \boxed{\frac{1}{4}(4x-19)^{-\frac{1}{2}}(4)}$ OR $\frac{1}{\sqrt{4x-19}}$

$(f^{-1})'(5) = \frac{1}{\sqrt{4(5)-19}} = 1$

(c) $(f^{-1})'(a) = \frac{1}{f'(f^{-1}(a))} = \frac{1}{f'(f^{-1}(5))}$, since $a=5$

1st find $f'(x) = 2x-5$

2nd find $f^{-1}(a) = f^{-1}(5)$

Find x such that $f(x)=5$.

$f(x) = 5$

$x^2-5x+11 = 5$

$x^2-5x+6 = 0$

$(x-3)(x-2) = 0$

$x=3$ or $x=2$

$x=2 \notin [\frac{5}{2},\infty)$, so $\boxed{x=3}$ $f^{-1}(5)=3$

Now, $\frac{1}{f'(f^{-1}(5))} = \frac{1}{f'(3)} = \frac{1}{2(3)-5} = \frac{1}{6-5} = \frac{1}{1} = 1$

$$\boxed{(f^{-1})'(5) = 1}$$

(B) Find derivative wrt x

(a) $y = 7\cdot 3^{\cos(x)} \longrightarrow y' = 7(-\sin(x))(\ln(3))(3^{\cos(x)})$ ← Stop Here is OK.

$= \boxed{-7\ln(3)\sin(x)\cdot 3^{\cos(x)} = y'}$ (Don't need to simplify)

(b) $y = \ln\left(\frac{\sqrt[3]{x^2-5x}}{\sin^3(x)}\right) = \frac{1}{3}\ln(x^2-5x) - 3\ln(\sin(x))$

$= \frac{1}{3}\ln(x(x-5)) - 3\ln(\sin(x))$

$= \frac{1}{3}\ln(x) + \frac{1}{3}\ln(x-5) - 3\ln(\sin(x))$

$\longrightarrow \boxed{y' = \frac{1}{3}\left(\frac{1}{x}\right) + \frac{1}{3}\left(\frac{1}{x-5}\right) - 3\left(\frac{\cos(x)}{\sin(x)}\right)} = \frac{1}{3}\left(\frac{2x-5}{x^2-5x}\right) - 3\cot(x)$

or $\frac{1}{3}\left(\frac{1}{x} + \frac{1}{x-5}\right) - 3\cot(x)$ or etc.

(c) $y = \log_5(x^2-3x) = \frac{1}{\ln(5)}\ln(x^2-3x) \longrightarrow$

$\boxed{y' = \frac{1}{\ln(5)}\left(\frac{2x-3}{x^2-3x}\right)}$

(d) $y = (\cos(x))^{x^2-3x} \longrightarrow$

$\ln(y) = (x^2-3x)\ln(\cos(x)) \longrightarrow$

$\frac{y'}{y} = (2x-3)\ln(\cos(x)) + (x^2-3x)\left(\frac{-\sin(x)}{\cos(x)}\right)$

$= (2x-3)\ln(\cos(x)) - (x^2-3x)\tan(x) \longrightarrow$

$\boxed{y' = \left((2x-3)\ln(\cos(x)) - (x^2-3x)\tan(x)\right)(\cos(x))^{x^2-3x}}$

9. $\frac{1}{2}$-life is 80yrs
Let $N(t)$ = amount of Radioactive Millsium in sample in grams, as a function of
t = number of years old the sample is.

Half-life is 80 years means

$$N(80) = \frac{1}{2}N(0)$$

Radioactive decay rate is proportional to the amount present.
This is a fancy way of saying that it's exponential decay

$\boxed{N(t) = N_0 e^{kt}}$. We find k & expect it must be negative.

$N_0 = N(0)$ = original amount of Millsium.

$N(80) = \frac{1}{2}N_0$ means

$$N_0 e^{80k} = \frac{1}{2}N_0 \Rightarrow$$

$$e^{80k} = \frac{1}{2} \Rightarrow$$

$$80k = \ln\left(\tfrac{1}{2}\right) = \ln(2^{-1}) = -\ln(2) \Rightarrow$$

$$\boxed{k = -\frac{\ln(2)}{80}}$$

So $\boxed{N(t) = N_0 e^{-\frac{\ln(2)}{80}t}}$

How old is a sample that has 12% of its original Millsium?

$$N(t) = N_0 e^{kt} = .12N_0$$

$$e^{kt} = .12$$

$$kt = \ln(.12)$$

$$t = \frac{\ln(.12)}{k} = \frac{\ln(.12)}{-\frac{\ln(2)}{80}} = \boxed{-\frac{80\ln(.12)}{\ln(2)} = t}$$

$\boxed{t \approx 244.7114951 \text{ yrs old}}$

Check

0 – 100%

80 – 50%

160 – 25%

240 – 12.5%

So $t \approx 245$ & 12% seems reasonable.

METHOD 2 $\frac{1}{2}$-life by powers of $\frac{1}{2}$.

$$N_0\left(\tfrac{1}{2}\right)^{kt} = N(t)$$

$$N_0\left(\tfrac{1}{2}\right)^{80k} = \tfrac{1}{2}N_0$$

$$\left(\tfrac{1}{2}\right)^{80k} = \tfrac{1}{2}$$

$$\log_2\left(\tfrac{1}{2}\right)^{80k} = \log_2\left(\tfrac{1}{2}\right) = \log_2\left(2^{-1}\right)$$

$$\log_2\left(2^{-80k}\right) = -1$$

$$-80k = -1$$

$$k = \frac{1}{80}$$

$$N(t) = N_0\left(\tfrac{1}{2}\right)^{\frac{1}{80}t}$$

$$N(t) \overset{SET}{=} .12N_0$$

$$N_0\left(\tfrac{1}{2}\right)^{\frac{1}{80}t} = .12N_0$$

$$\left(\tfrac{1}{2}\right)^{\frac{1}{80}t} = .12$$

$$\log_2\left(\tfrac{1}{2}\right)^{\frac{1}{80}t} = \log_2\left(\left(2^{-1}\right)^{\frac{1}{80}t}\right) = \log_2\left(2^{-\frac{1}{80}t}\right) = \log_2(.12)$$

$$-\frac{1}{80}t = \log_2(.12)$$

$$\Rightarrow t = -80\log_2(.12) = \frac{-80\ln(.12)}{\ln(2)}$$

$\approx$ same.