1. (10 pts) Sketch the graph of the polynomial $f(x) = 3x^3 - 5x^2 - 48x + 80$.

$\mathcal{D} = (-\infty, \infty)$

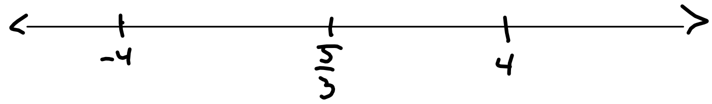
$3x^3 \quad \swarrow \cdots \nearrow$

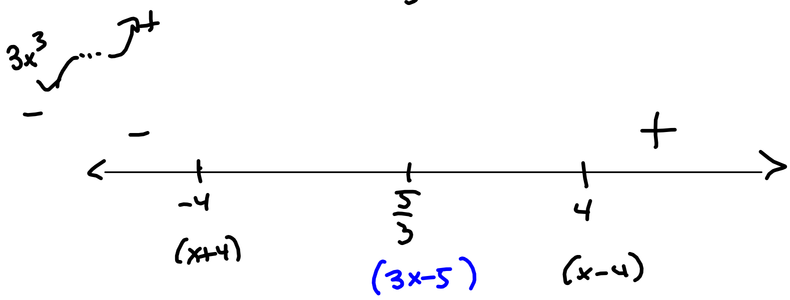
No Asymptotes

Observe that it factors by grouping:

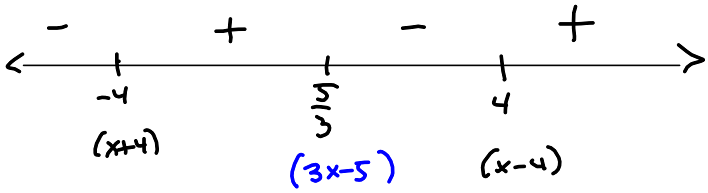
$$f(x) = 3x^3 - 5x^2 - 16\cdot 3x + 16\cdot 5$$
$$= x^2(3x-5) - 16(3x-5)$$
$$= (3x-5)(x^2-16)$$
$$= (3x-5)(x-4)(x+4) \overset{\text{SET}}{=} 0 \longrightarrow$$

$x = \pm 4, \frac{5}{3}$

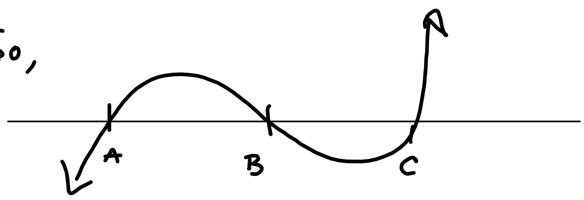




All the powers/multiplicities are '1.' '1' is odd. Sign Changes



So,



$A = (-4, 0)$

$B = (\frac{5}{3}, 0)$

$C = (4, 0)$

$f'(x) = 9x^2\ 10x - 48$ $9 = 3^2$ $48 \to 2^4 \cdot 3$ $-2\cdot2\cdot2\cdot2\cdot3\cdot3\cdot3$

$a = 9,\ b = -10,\ c = -48$

M1

$b^2 - 4ac = 10^2 - 4(9)(-48)$

$= 100 + 36(48)$

$= 100 + 1728$

$= 1828$ is not a perfect square.

$$\begin{array}{r} 48 \\ \times\ 36 \\ \hline 288 \\ 1440 \\ \hline 1728 \end{array}$$

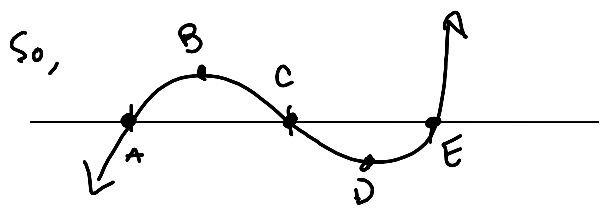
$$\begin{array}{r|l} 2 & 1828 \\ 2 & 914 \\ & 457 \end{array}$$

457 is prime.

So $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{10 \pm 2\sqrt{457}}{2(9)} = \dfrac{5 \pm \sqrt{457}}{9}$

$\dfrac{5 - \sqrt{457}}{9}$ ≈ -1.819728703 $\dfrac{5 + \sqrt{457}}{9}$ ≈ 2.930839815

So,



$A = (-4, 0)$

$B = \left(\dfrac{5-\sqrt{457}}{9},\ f\left(\dfrac{5-\sqrt{457}}{9}\right)\right)$ MAX

$C = \left(\dfrac{5}{3}, 0\right)$

$D = \left(\dfrac{5+\sqrt{457}}{9},\ f\left(\dfrac{5+\sqrt{457}}{9}\right)\right)$

$E = (4, 0)$

Points *B* and *D* aren't much fun. Probably the easiest way is to get a decimal result for

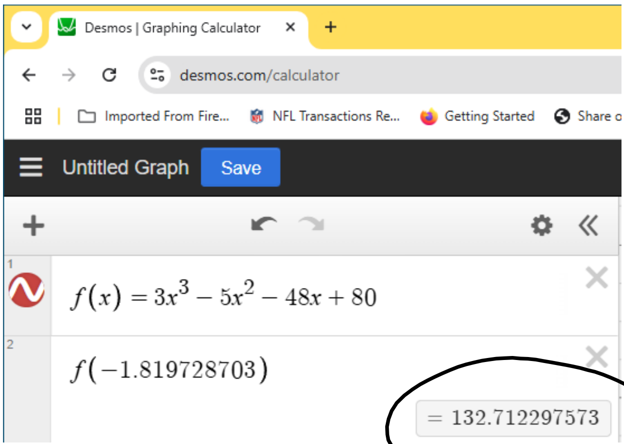
$\dfrac{5-\sqrt{457}}{9}$ ≈ -1.819728703

& $\dfrac{5+\sqrt{457}}{9}$ ≈ 2.930839815

If you're using a graphing calculator, you can enter $f(x)$ into Y1 and evaluate it at each of those decimal values.

I didn't specify a precision, so I'm looking to see that you are correct as far as you take it.

Another strategy is to enter $f(x)$ into Desmos:



$f\left(\frac{5-\sqrt{457}}{9}\right)$, approximately

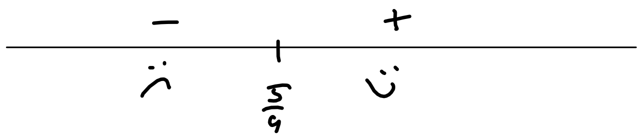
The graph isn't shown, but it's not much help, because of the square viewing window.

We just see what looks like something vertical, It's hard to capture the essence of the shape, this way.

A graphing calculator allows you to make tall, skinny windows to flatten things out, but we don't need that picture, since we know the general shape and it's just a matter of labeling where certain things happen.

There's one more level to analyze: The 2nd Derivative:

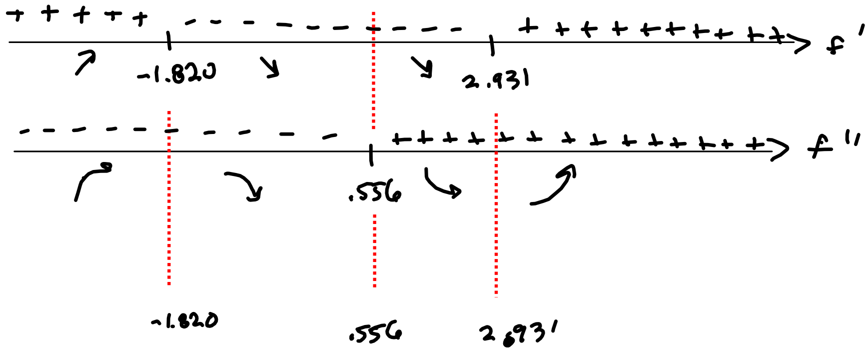
$f''(x) = 18x - 10 \overset{SET}{=} 0 \longrightarrow 18x = 10 \longrightarrow x = \frac{10}{18} = \frac{5}{9}$

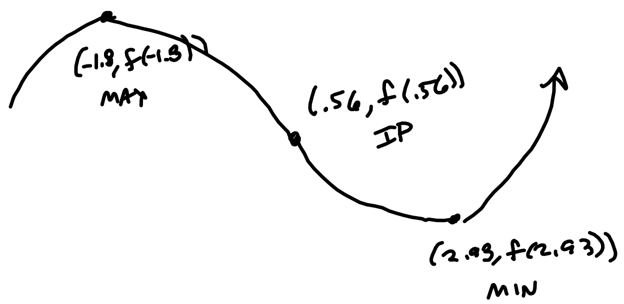


$\approx$ 0.555555555556

$18x-10$

Now put it all together with the sign patterns of the 1st and 2nd derivative interacting:

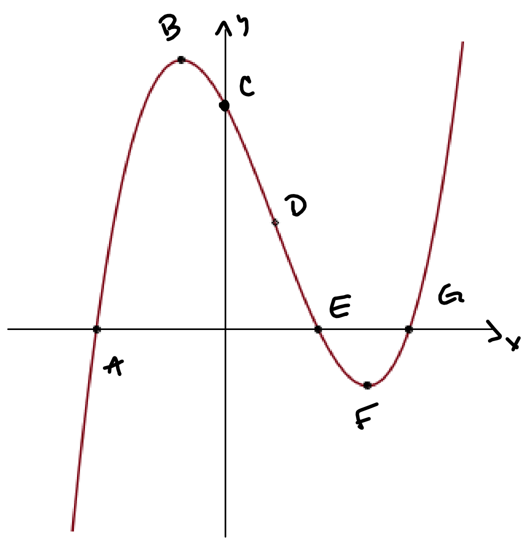




$f(-1.8\ldots) \approx 132.712297573$

$f(.55\ldots) \approx 52.30452675 \longrightarrow$ Inflection Point is ABOVE the x-axis.

$f(2.9\ldots) \approx -28.10324408$



$A = (-4, 0)$ x-int

$B \approx (-1.820, 132.712)$ MAX

$C = (0, 80)$ y-int

$D \approx (.556, 52.305)$ I.P.

$E = (\frac{5}{3}, 0) \approx (1.667, 0)$ x-int

$F \approx (2.931, -28.103)$ MIN

$G = (4, 0)$

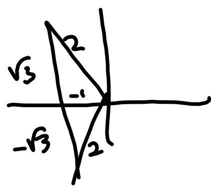
② $f(x) = \sin(2x) + x \overset{\text{SET}}{=} 0 \Rightarrow x = 0$

$f'(x) = 2\cos(2x) + 1 \overset{\text{SET}}{=} 0 \Rightarrow$

$2\cos(2x) = -1$

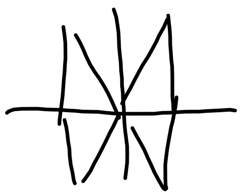
$\cos(2x) = -\frac{1}{2}$ Looking $\forall x \in [0, 2\pi)$

$\Rightarrow 2x \in [0, 4\pi)$



$2x = \pi - \frac{\pi}{3} = \frac{2\pi}{3}, \; \pi + \frac{\pi}{3} = \frac{4\pi}{3}, \; \frac{2\pi}{3} + 2\pi = \frac{8\pi}{3}, \; \frac{4\pi + 6\pi}{3} = \frac{10\pi}{3}$

$\Rightarrow x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$



$f\left(\frac{\pi}{3}\right) = \sin\left(2\left(\frac{\pi}{3}\right)\right) + \frac{\pi}{3} = \sin\left(\frac{2\pi}{3}\right) + \frac{\pi}{3} = \frac{\sqrt{3}}{2} + \frac{\pi}{3}$

$f\left(\frac{2\pi}{3}\right) = \sin\left(\frac{4\pi}{3}\right) + \frac{2\pi}{3} = -\frac{\sqrt{3}}{2} + \frac{2\pi}{3}$

$f\left(\frac{4\pi}{3}\right) = \sin\left(\frac{8\pi}{3}\right) + \frac{4\pi}{3} = \sin\left(\frac{2\pi}{3}\right) + \frac{4\pi}{3} = \frac{\sqrt{3}}{2} + \frac{4\pi}{3}$

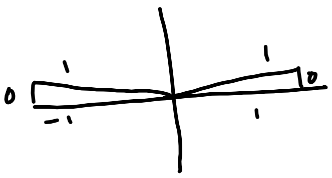
$f\left(\frac{5\pi}{3}\right) = \sin\left(\frac{10\pi}{3}\right) + \frac{5\pi}{3} = \sin\left(\frac{4\pi}{3}\right) + \frac{5\pi}{3} = -\frac{\sqrt{3}}{2} + \frac{5\pi}{3}$

$f(x) = \sin(2x) + x \rightarrow$

$f'(x) = 2\cos(2x) + 1 \rightarrow$

$f''(x) = -4\sin(2x) \overset{\text{SET}}{=} 0 \rightarrow$

$\sin(2x) = 0$



$2x = 0, \pi, 2\pi, 3\pi, 4\pi$

$x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$

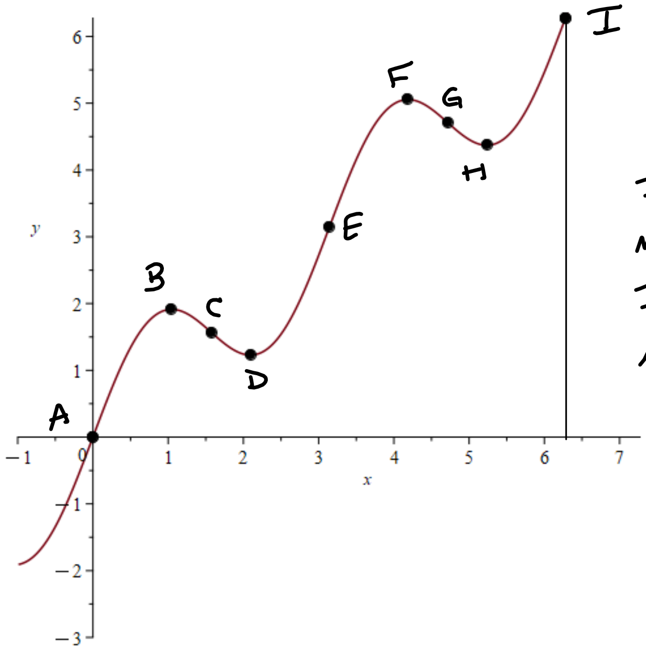
$f(0) = 0$

$f(\frac{\pi}{2}) = \sin(\pi) + \frac{\pi}{2} = \frac{\pi}{2}$

$f(\pi) = \pi$

$f(\frac{3\pi}{2}) = \sin(3\pi) + \frac{3\pi}{2} = \frac{3\pi}{2}$

$f(2\pi) = 2\pi$



IP $A = (0, 0)$

MAX $B = \left(\frac{\pi}{3}, \frac{\pi}{3} + \frac{\sqrt{3}}{2}\right) \approx (1.05, 1.91)$

IP $C = \left(\frac{\pi}{2}, \frac{\pi}{2}\right) \approx (1.57, 1.57)$

MIN $D = \left(\frac{2\pi}{3}, \frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right) \approx (2.09, 1.23)$

IP $E = \left(\frac{3\pi}{2}, \frac{3\pi}{2}\right) \approx (4.71, 4.71)$

MAX $F = \left(\frac{4\pi}{3}, \frac{4\pi}{3} + \frac{\sqrt{3}}{2}\right) \approx (4.19, 5.05)$

IP $G = (\pi, \pi) \approx (3.14, 3.14)$

MIN $H = \left(\frac{5\pi}{3}, \frac{5\pi}{3} - \frac{\sqrt{3}}{2}\right) \approx (5.24, 4.37)$

IP $I = (2\pi, 2\pi) \approx (6.28, 6.28)$

3 $R(x) = \dfrac{x^2-3x-28}{x-1} = \dfrac{(x-7)(x+4)}{x-1} =$

$\mathcal{D} = \mathbb{R} \setminus \{1\}$. Vert. Asymp.: $\boxed{x=1 \quad \text{V.A}}$

$R(x) = 0 \Longrightarrow x = -4, 7$

x-int: $(-4,0), (7,0)$

y-int: $(0,28)$

Slant Asymptote:

$$\begin{array}{r|rrr} 1 & 1 & -3 & -28 \\ & & 1 & \\ \hline & 1 & -2 & \end{array}$$

$\rightarrow \boxed{y = x-2 \text{ is S.A.}}$

$$R'(x) = \frac{(2x-3)(x-1) - (x^2-3x-28)}{(x-1)^2} = \frac{2x^2-5x-3x+3-x^2+3x+28}{(x-1)^2}$$

$$= \frac{x^2-2x+31}{(x-1)^2}$$. V.A.: $x=1$ No sign change

$x^2-2x+1^2-1+31 = (x-1)^2+30 = 0 \longrightarrow$ No real soln.

$\boxed{R'(x) > 0 \text{ on its domain}}$

$$R''(x) = \frac{(2x-2)(x-1)^2 - (x^2-2x+31)(2(x-1))}{(x-1)^4}$$

$$= \frac{(2x-2)(x-1) - 2(x^2-2x+31)}{(x-1)^3}$$

$$= \frac{2x^2-4x+2-2x^2+4x-62}{(x-1)^3} = \frac{-60}{(x-1)^3}$$

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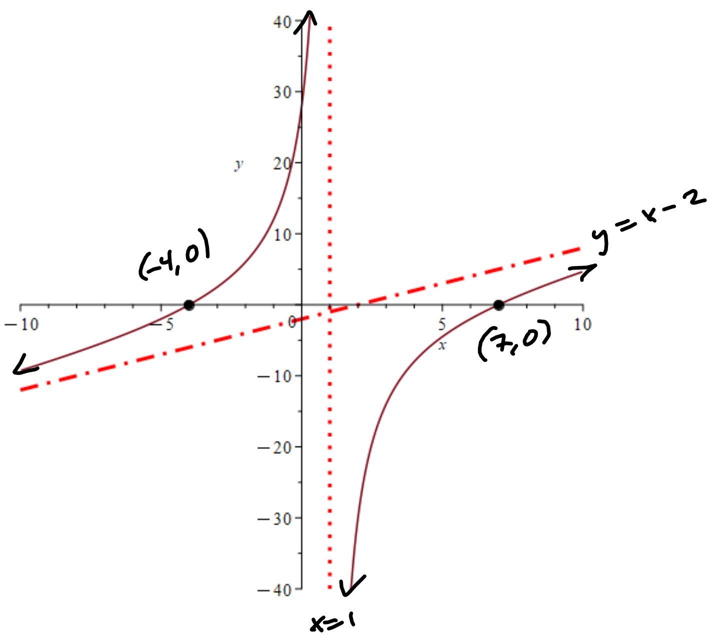
$\not{\mathcal{D}}$


(-4,0)
(7,0)
y = x - 2
x=1