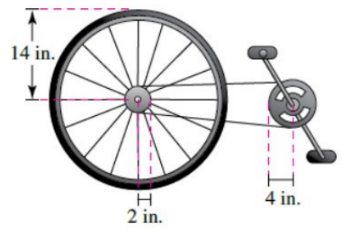
Writing Project 0 - Trig Review

1. (10 pts) The radii of the pedal sprocket, the wheel sprocket, and the rear wheel of the bicycle are 4 inches, 2 inches, and 14 inches, respectively. A cyclist is pedaling at a rate of 1 revolution per second. Find the speed of the bicycle in feet per second and miles per hour.

   See figure on the right.



This is an "arc-length per unit time" question.

Recall, arc length $= s = r\theta$

we'll build an $\frac{r\theta}{t}$ function

$$\left(\frac{1 \text{ rev front}}{\text{sec}}\right)\left(\frac{4 \text{ revs rear}}{2 \text{ revs front}}\right)^{*}\left(\frac{2\pi \text{ radians}}{1 \text{ rev}}\right) = \text{Angular speed of rear}$$

*Rear sprocket spins 4 times every time the front spins once, because its radius is 2" & front is 4".

now to see how fast you're moving on the ground, use $s = r\theta$, i.e., multiply by radius of rear wheel:

$$\frac{(2)(2\pi)(14 \text{ inches})}{\text{sec.}} = \frac{56\pi \text{ inches}}{\text{sec.}}$$

we want ft/sec so

$$\left(\frac{56\pi \text{ in}}{\text{sec}}\right)\left(\frac{1 \text{ foot}}{12 \text{ in}}\right) = \frac{56\pi}{12}\frac{\text{ft}}{\text{sec}} = \frac{28\pi}{6}\frac{\text{ft}}{\text{s}} = \boxed{\frac{14\pi}{3}\frac{\text{ft}}{\text{sec}}}$$

≈ 14.66076572 ft/sec

Convert to $\frac{\text{mi}}{\text{hr}}$

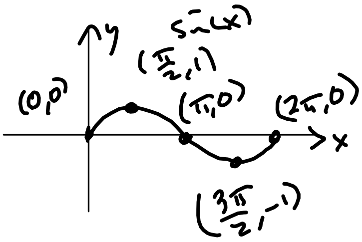
method 1:

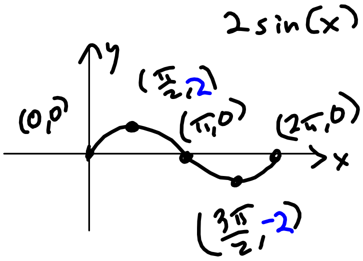
$$\left(\frac{14\pi}{3}\frac{\text{ft}}{\text{s}}\right)\left(\frac{1 \text{ mi}}{5280 \text{ ft}}\right)\left(\frac{60 \text{ sec}}{1 \text{ min}}\right)\left(\frac{60 \text{ min}}{1 \text{ hr}}\right) = \frac{(14\pi)(3600)}{(3)(5280)} = \boxed{\frac{35\pi}{11}\frac{\text{mi}}{\text{hr}}}$$

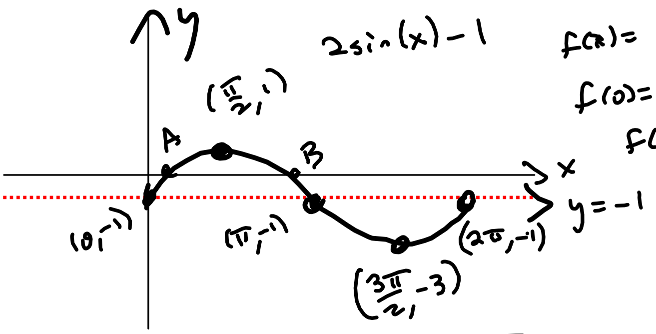
method 2: $\frac{88 \text{ ft}}{\text{s}} = \frac{60 \text{ mi}}{\text{hr}}$

$$\left(\frac{14\pi \text{ ft}}{3 \text{ sec}}\right)\left(\frac{60 \frac{\text{mi}}{\text{hr}}}{88 \frac{\text{ft}}{\text{s}}}\right) = \boxed{\frac{35\pi}{11}\frac{\text{mi}}{\text{hr}}} \approx 9.99597625 \frac{\text{mi}}{\text{hr}}$$

2. (10 pts) Sketch the graph of $f(x) = 2\sin(x) - 1$.



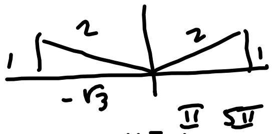




$f(x) = 2\sin(x) - 1$

$f(0) = -1$

$f(x) = 0 \longrightarrow 2\sin(x) = 1 \longrightarrow \sin(x) = \frac{1}{2} \longrightarrow$



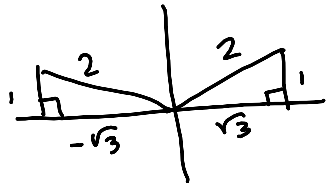
$x = \frac{\pi}{6}, \frac{5\pi}{6}$

$\boxed{A = \left(\frac{\pi}{6}, 0\right),\ B = \left(\frac{5\pi}{6}, 0\right)}$

3 a 5pts Solve $\sin(3x) = \frac{1}{2}$ on $[0, 2\pi)$

Want $x \in [0, 2\pi)$, so want $3x \in [0, 6\pi)$

$\sin(3x) = \frac{1}{2}$



$3x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{6} + 2\pi = \frac{13\pi}{6}, \frac{5\pi}{6} + 2\pi = \frac{17\pi}{6},$

$\frac{25\pi}{6}$, and $\frac{29\pi}{6}$

If we go another 2π, we've gone too far!

$\frac{25\pi}{6} + \frac{12\pi}{6} = \frac{37\pi}{6} = 6\pi + \frac{\pi}{6}$. see? Too big

Need it less than 6π, so x will be less than 2π when we divide by 3.

This means $\boxed{x \in \left\{ \frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}, \frac{25\pi}{18}, \frac{29\pi}{18} \right\}}$

b 5pts

$3x = \frac{\pi}{6} + 2n\pi$

$x = \frac{\pi}{18} + \frac{2n\pi}{3}$

$3x = \frac{5\pi}{6} + 2n\pi$

$x = \frac{5\pi}{18} + \frac{2n\pi}{3}$

WebAssign Style: $\frac{\pi}{18} + \frac{2n\pi}{3}, \frac{5\pi}{18} + \frac{2n\pi}{3}$

Solution set: $x \in \left\{ y \mid y = \frac{\pi}{18} + \frac{2\pi n}{3} \text{ OR } y = \frac{5\pi}{18} + \frac{2n\pi}{3} \right\}$

OR $\left\{ y + \frac{2n\pi}{3} \mid y = \frac{\pi}{18} \text{ OR } y = \frac{5\pi}{18} \right\}$

OR $\left\{ y + \frac{2n\pi}{3} \mid y \in \left\{ \frac{\pi}{18}, \frac{5\pi}{18} \right\} \right\}$

4. (10 pts) Construct a cosine function that models the periodic data from the temperature table for Las Vegas on the right. Your model should have a period of 12 months.

Month, t	Las Vegas, L	International Falls, I
1	57.1	13.8
2	63.0	22.4
3	69.5	34.9
4	78.1	51.5
5	87.8	66.6
6	98.9	74.2
7	104.1	78.6
8	101.8	76.3
9	93.8	64.7
10	80.8	51.7
11	66.0	32.5
12	57.3	18.1

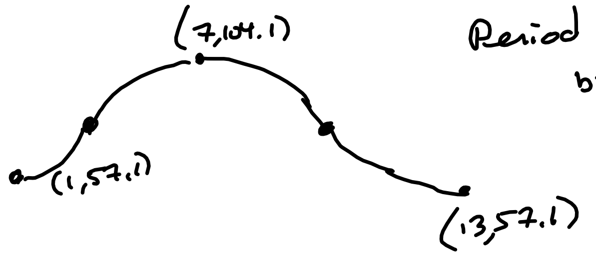
Nathan asks "Which city's temperature data are we to model?" I reply: "Question was poorly posed. Either one will do. Because I didn't specify, I need to do both."

(1, 57.1) is low.

(7, 104.1) is high.

cosine is highest @ $t=7$, so

$a\cos(b(t-7))+d$



Period is 12

$bt = 2\pi$ when $t=12$:

$12b = 2\pi$

$b = \frac{2\pi}{12} = \frac{\pi}{6}$

$a\cos(\frac{\pi}{6}(t-7))+d$

Midline: $\frac{104.1+57.1}{2} = \frac{161.2}{2} = 80.6$

$a\cos(\frac{\pi}{6}(t-7))+80.6$

Vegas Amplitude: $\frac{104.1-57.1}{2} = \frac{47}{2} = 23.5$

$\boxed{f(t) = 23.5\cos(\frac{\pi}{6}(t-7))+80.6}$

Int'l Falls

$a\cos(\frac{\pi}{6}(t-7))+d$ Same hot month / cold month
Same period

$\frac{\text{High}+\text{Low}}{2}$ = midline =

$\frac{78.6+13.8}{2} = \frac{92.4}{2} = 46.2$

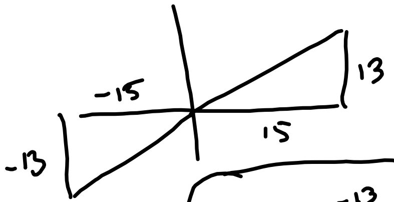
$\frac{\text{HIGH}-\text{LOW}}{2}$ = Amplitude $= \frac{78.6-13.8}{2} = \frac{64.8}{2} = 32.4$, so

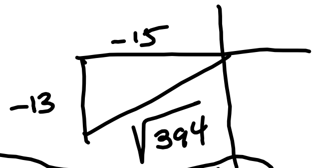
$\boxed{f(t) = 32.4\cos(\frac{\pi}{6}(t-7))+46.2}$

5. (10 pts) Given that $\tan(\beta) = \dfrac{13}{15}$ and $\sin(\beta) < 0$, find the exact value of the other 5 trigonometric functions.

$\tan\beta = \frac{13}{15}$

AND $\sin(\beta) < 0 \Longrightarrow$





is the one

$13^2 + 15^2 = 169 + 225$

$= 394$

$2 \overline{)394}$ 197 Doesn't simplify,

$\sqrt{394}$ doesn't, I mean.

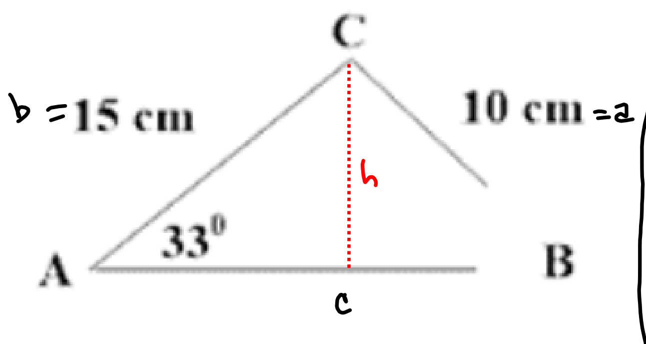
$$\boxed{\begin{array}{ll} \sin\beta = \dfrac{-13}{\sqrt{394}} & \csc\beta = -\dfrac{\sqrt{394}}{13} \\ \cos\beta = \dfrac{-15}{\sqrt{394}} & \sec\beta = -\dfrac{\sqrt{394}}{15} \\ \tan\beta = \dfrac{13}{15} & \cot\beta = \dfrac{15}{13} \end{array}}$$

6. Suppose $A = 33^0$, $a = 10$ cm, and $b = 15$ cm.

   a. (5 pts) Show that there are two solutions to this triangle, before solving the triangle.

   b. (5 pts) (Law of Sines) For one, B will be acute. That's the first solution that the Law of Sines will produce. For the other, B will be obtuse. Round final answers to 3 decimal places. See figure on the right.

(a) (5pts) Height of triangle is h



$$\frac{h}{15} = \sin 33^0$$

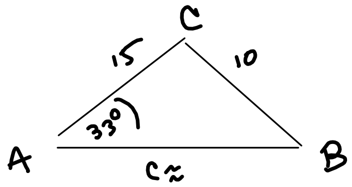
$h = 15 \sin 33^0 \approx 8.169585525$

$8.17 \approx h < a = 10$, so at least one solution.

$h < b = 15 \Rightarrow$ 2 sol'ns

198^0

Sol'n 1: B is acute



$$\frac{\sin B}{b} = \frac{\sin B}{15} = \frac{\sin A}{a} = \frac{\sin 33^\circ}{10}$$

$$\Rightarrow \sin B = \frac{15 \sin 33^\circ}{10} \approx .8169585525$$

$$\Rightarrow B = \arcsin(.816\ldots) \approx 54.78148$$

is acute B.

For obtuse B: $180^\circ - 54.78148198^\circ \approx 125.218518^\circ \approx$ Obtuse B

$$\boxed{\begin{array}{l} B_a \approx 54.781^\circ \\ C_a \approx 92.219^\circ \\ c_a \approx 18.347 \text{ cm} \end{array}} \qquad \boxed{\begin{array}{l} B_o \approx 125.219^\circ \\ C_o \approx 21.781^\circ \\ c_o \approx 6.183 \text{ cm} \end{array}}$$

$C = 180^\circ - A - B \approx$

- B Acute: $180^\circ - 33^\circ - \cancel{54.78148198^\circ} \approx 92.21851802^\circ \approx C \ (B_A)$
- B obtuse: $180^\circ - 33^\circ - 125.218518^\circ \approx 21.781482^\circ \approx C \ (B_o)$

Find side c:

B ACUTE: $\frac{c}{\sin C} = \frac{a}{\sin A} \Rightarrow c = \frac{a \sin C}{\sin A} \approx \frac{10 \sin(92.21851802^\circ)}{\sin(33^\circ)}$

$\approx 18.3470024 \approx c \ (B_A)$

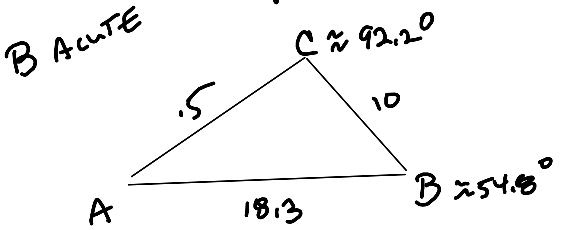
B OBTUSE: $c = \frac{10 \sin(21.781482^\circ)}{\sin(33^\circ)} \approx 6.813094648 \approx c \ (B_O)$

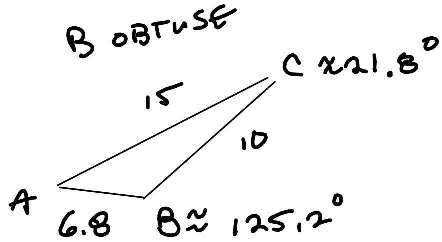
B_o

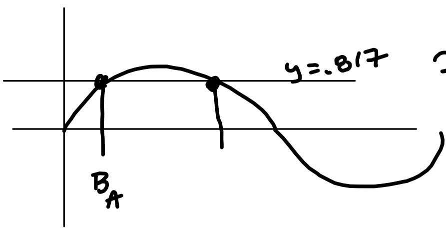
$B_o = 180^\circ - B_A$

$$\boxed{\begin{array}{l} B_a \approx 54.781^\circ \\ C_a \approx 92.219^\circ \\ c_a \approx 18.347 \text{ cm} \end{array}} \qquad \boxed{\begin{array}{l} B_o \approx 125.219^\circ \\ C_o \approx 21.781^\circ \\ c_o \approx 6.183 \text{ cm} \end{array}}$$

2 pics:







2 spots on $[0,\pi)$ where $\sin B = .817$