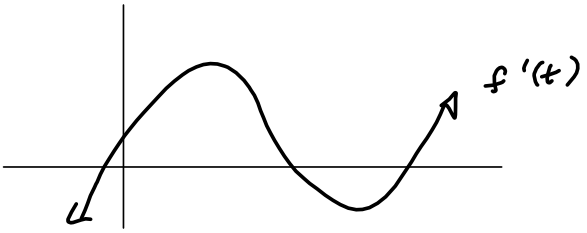
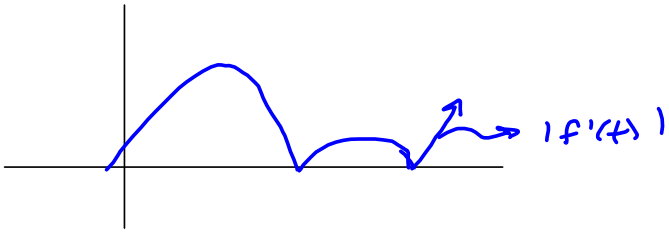
$f'(t)$ is a rate function

$$\int_a^b f'(t)\,dt = \text{net change} = f(b) - f(a)$$

$$\int_a^b |f'(t)|\,dt = \text{Total change}$$



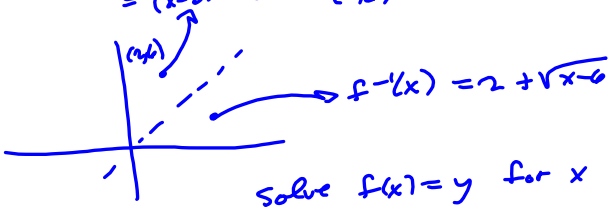


Something Like #7 on $[2,\infty)$

$f(x) = x^2-4x+10$

$= x^2-4x+2^2-4+10$

$= (x-2)^2+6$ (2,6)



Solve $f(x)=y$ for x:

$(x-2)^2+6 = y$

$(x-2)^2 = y-6$

$x-2 = \pm\sqrt{y-6}$

$x = 2\pm\sqrt{y-6}$

$f^{-1}(x) = 2+\sqrt{x-6}$

$= 2+(x-6)^{\frac{1}{2}}$

$(f^{-1})'(x) = \frac{1}{2}(x-6)^{-\frac{1}{2}}(1) = \frac{1}{2\sqrt{x-6}}$

$(f^{-1})'(106) = \frac{1}{2\sqrt{106-6}} = \frac{1}{20}$ ✓

Finding $f^{-1}(x)$ with quadratic formula

$x^2-4x+10 = y$

$x^2-4x+10-y = 0$

$a=1, b=-4, c=10-y$

$b^2-4ac = 16-4(1)(10-y)$

$= 16-4(10-y)$

$= 16-40+4y$

$= 4y-24 = 4(y-6)$

$x = \frac{4\pm\sqrt{4(y-6)}}{2}$

$= \frac{4\pm 2\sqrt{y-6}}{2}$

$= 2\pm\sqrt{y-6}$

So, $f^{-1}(x) = 2+\sqrt{x-6}$

$10-c=96 \Rightarrow c = \cancel{86}$

Find $(f^{-1})'(106)$

$(f^{-1})'(106) = 2+\sqrt{106-6} = 2+\sqrt{100} = 2+10 = 12 = (f^{-1})'(106)$

This is $f^{-1}(106)$ not $(f^{-1})'(106)$

No.

The formula: $(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))} = \frac{1}{2x-4}$

$f'(x) = 2x-4$

$f^{-1}(106)$:

$f(x) = 106$

$x^2-4x+10 = 106$

$x^2-4x-96 = 0$

$(x-12)(x+8) = 0$

$x = 12$ or $x=-8 \notin [2,\infty)$

$x = 12 = f^{-1}(106)$

$f'(12) = 2(12)-4 = 20$

$\frac{1}{f'(12)} = \frac{1}{2(12)-4} = \frac{1}{20}$?!

Yes!

$f(x) = x^2+4x+10$

Find $(f^{-1})'(106)$

$$\frac{1}{f'(f^{-1}(x))} = \frac{1}{f'(f^{-1}(106))}$$

$f(x) = 106$

$x^2+4x+10 = 106$

$x^2+4x-96 = 0$

$(x-12)(x+8) = 0$

$x = 12 = f^{-1}(106)$

$$\frac{1}{f'(f^{-1}(106))} = \frac{1}{2(12)+4} = \frac{1}{20}$$

Find $f^{-1}(x)$ by completing the square

$f(x) = y$

$x^2+4x+10 = y$

$x^2+4x = y-10$

$x^2+4x+2^2 = y-10+4$

$(x+2)^2 = y-6$

$x = -2 \pm \sqrt{y-6}$ $\Rightarrow$ $f^{-1}(x) = -2+(x-6)^{\frac{1}{2}}$

$(f^{-1})'(x) = \frac{1}{2}(x-6)^{-\frac{1}{2}} = (f^{-1})'(x)$

$(f^{-1})'(106) = \frac{1}{2\sqrt{100}} = \frac{1}{20}$ Finally!