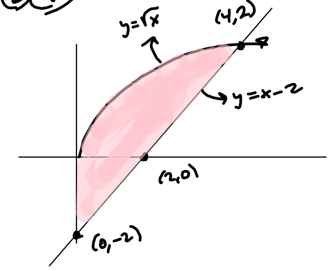
② Area bdd by $y=\sqrt{x}$, $y=x-2$, $x=0$

ⓐ (5 pts) sketch



$\sqrt{x} = x-2$

$x = (x-2)^2 = x^2-4x+4$

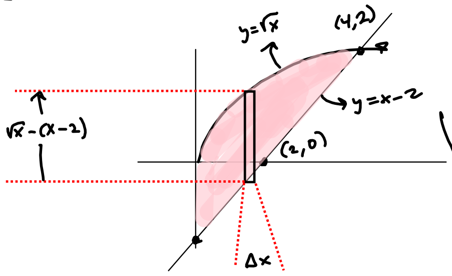
$\Rightarrow x^2-5x+4=0 \Rightarrow$

$(x-1)(x-4)=0 \Rightarrow$

$x \in \{1,4\}$. $x=1$ is extraneous

So $x=4, y=\sqrt{4}=2 \Rightarrow (4,2)$

ⓑ Show representative rectangle & write the integral for the area.



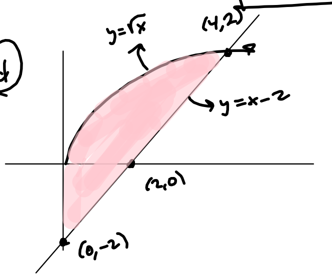
$$\text{Area} = \int_0^4 (\text{upper}-\text{lower})\,dx$$

$$\boxed{= \int_0^4 (\sqrt{x}-(x-2))\,dx}$$

ⓒ $= \int_0^4 (x^{\frac{1}{2}}-x+2)\,dx = \left[\frac{2}{3}x^{\frac{3}{2}} - \frac{x^2}{2} + 2x\right]_0^4 = \left[\frac{2}{3}(4)^{\frac{3}{2}} - \frac{4^2}{2} + 2(4) - 0\right]$

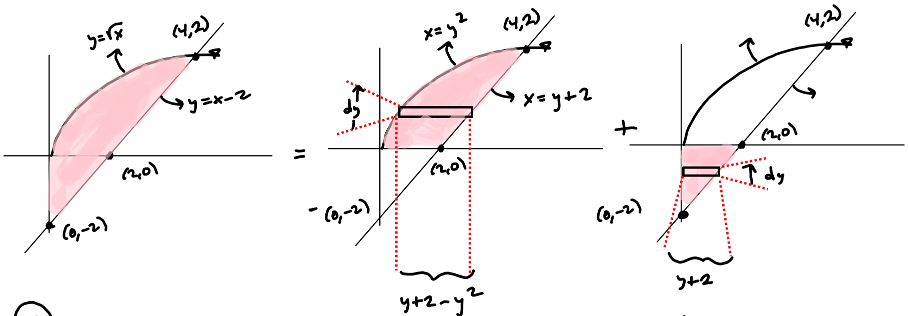
$= \frac{2}{3}(8) - 8 + 8 = \boxed{\frac{16}{3} = \text{Area}}$

ⓓ



ⓔ Write integral (actually 2 integrals) and show (a) representative rectangle(s).

Different "left" function for $y \in [-2,0]$ and $y \in [0,2]$:



ⓔ

$$\int_0^2 (y+2-y^2)\,dy + \int_{-2}^0 (y+2)\,dy$$

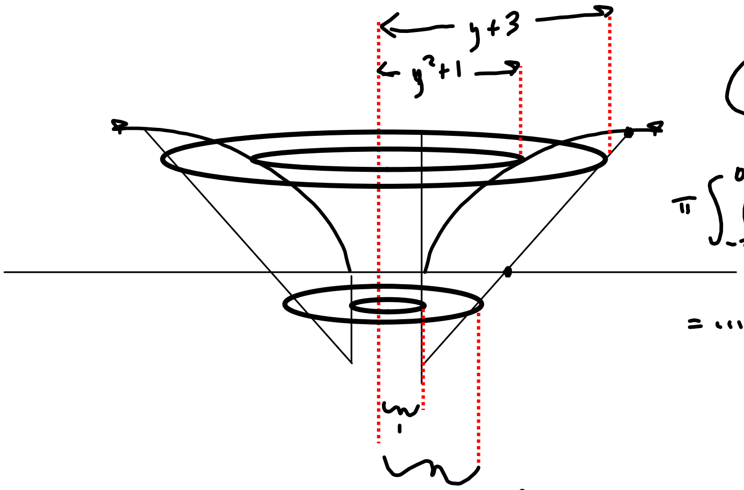
ⓕ

$$= \left[\frac{y^2}{2}+2y-\frac{y^3}{3}\right]_0^2 + \left[\frac{y^2}{2}+2y\right]_{-2}^0$$

$= \left(\frac{2^2}{2}+2(2)-\frac{2^3}{3}\right) - (0) + (0) - \left(\frac{(-2)^2}{2}+2(-2)\right)$

$= 2+4-\frac{8}{3} - \left(\frac{4}{2}-4\right) = 6-\frac{8}{3}-(2-4) = 6-\frac{8}{3}+2 = 8-\frac{8}{3} = \frac{24-8}{3} = \boxed{\frac{16}{3}}$

(g) $\frac{16}{3}$ for both!



$y+3$ (from $y+2-(-1)$)

(h)

Volume =

$$\pi\int_{-2}^{0}\left((y+3)^2-1^2\right)dy+\pi\int_{0}^{2}\left((y+3)^2-(y^2+1)^2\right)dy$$

$$= \dots = \frac{128\pi}{5}$$

$$\int e^u \, du = e^u + C$$

$$\int (e^{x^2-2x})(2x-2)\,dx = \int e^u \, du = e^u + C = \boxed{e^{x^2-2x} + C}$$

Let $u = x^2 - 2x$

Then $du = (2x-2)\,dx \Longrightarrow dx = \dfrac{du}{2x-2}$

$$\int e^{x^2-2x}(2x-2)\,dx = \int e^u (2x-2)\left(\frac{du}{2x-2}\right) = \int e^u \, du = \text{etc.}$$