WP #3

(1) (5 pts) Evaluate $\int_{-1}^{2}(x^2-3x)\,dx$ by the limit definition:

$[a,b]=[-1,2] \Rightarrow$

$\Delta x = \frac{b-a}{n} = \frac{2-(-1)}{n} = \frac{3}{n}$

$x_k = a+k(\Delta x) = -1+k\left(\frac{3}{n}\right) = -1+\frac{3k}{n}$

$f(x_k) = x_k^2 - 3x_k = \left(-1+\frac{3k}{n}\right)^2 - 3\left(-1+\frac{3k}{n}\right)$

$= 1-2\left(\frac{3k}{n}\right)+\left(\frac{3k}{n}\right)^2+3-\frac{9k}{n}$

$= 1-\frac{6k}{n}+\frac{9k^2}{n^2}+3-\frac{9k}{n} \longrightarrow$

$= 4-\frac{15k}{n}+\frac{9}{n^2}k^2 \Longrightarrow$

$$\int_{-1}^{2} f(x)\,dx \approx \Delta x \sum_{k=1}^{n} f(x_k) = \frac{3}{n}\sum_{k=1}^{n}\left(4-\frac{15}{n}k+\frac{9}{n^2}k^2\right)$$

$$= \frac{3}{n}\sum 4 - \frac{3}{n}\cdot\frac{15}{n}\sum_{k=1}^{n} k + \frac{3}{n}\cdot\frac{9}{n^2}\sum_{k=1}^{n} k^2$$

$$= \boxed{\frac{3}{n}\cdot 4n - \frac{45}{n^2}\left[\frac{n^2+\sim}{2}\right] + \frac{27}{n^3}\left[\frac{n^3+\sim}{3}\right]}$$

$n\to\infty \longrightarrow 12-\frac{45}{2}+9 = 21-\frac{45}{2} = \frac{42}{2}-\frac{45}{2} = \boxed{-\frac{3}{2}}$

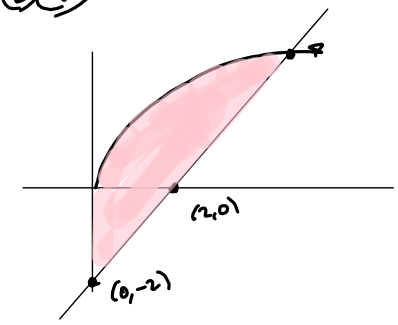
Check w/ FTC II!

$$\int_{-1}^{2}(x^2-3x)\,dx = \left[\frac{x^3}{3}-\frac{3x^2}{2}\right]_{-1}^{2} = \frac{2^3}{3}-\frac{3(2)^2}{2}-\left[\frac{(-1)^3}{3}-\frac{3(-1)^2}{2}\right]$$

$$= \frac{8}{3}-\frac{12}{2}+\frac{1}{3}+\frac{3}{2} = \frac{9}{3}-\frac{9}{2} = \frac{18-27}{6} = -\frac{9}{6} = \boxed{-\frac{3}{2}}$$ ✓

(2) Area bdd by $y=\sqrt{x}$, $y=x-2$, $x=0$

(2) (5pts) sketch



$\sqrt{x} = x-2$

$x = (x-2)^2 = x^2-4x+4$