Today, 4.2.

4.3 is on the immediate horizon.

I notice that there is a shortage of example videos for 4.2 and 4.3

The thing that I see a bit lacking in 4.2:

Emphasize

$$\sum_{k=0}^{n} k = \frac{n(n+1)}{2} = \frac{n^2+n}{2} = \frac{n^2 + \text{lower\_degree}}{2} = \frac{n^2 + \sim}{2}$$

$$\sum_{k=0}^{n} k^2 = \frac{n(n+1)(2n+1)}{6} = \frac{2n^3+3n^2+n}{6} = \frac{2n^3+\text{lower}}{6} = \frac{n^3+\sim}{3}$$

$$\sum_{k=0}^{n} k^3 = \frac{(n(n+1))^2}{2^2} = \frac{(n^2+n)^2}{4} = \frac{n^4+2n^3+n^2}{4} = \frac{n^4+\sim}{4}$$

$$\int_2^5 x^2\,dx \qquad \frac{b-a}{n} = \frac{5-2}{n} = \frac{3}{n} = \Delta x$$

$$x_k = 2 + k\Delta x$$

$$= 2 + k\cdot\frac{3}{n} = 2 + \frac{3k}{n} = \frac{3k+2n}{n}$$

$$\Delta x \sum_{k=1}^{n} f(x_k)$$

$$= \frac{3}{n}\sum_{k=1}^{n} x_k^2 = \frac{3}{n}\sum_{k=1}^{n}\left(\frac{3k}{n}+2\right)^2 = \frac{3}{n}\sum_{k=1}^{n}\left(\left(\frac{3k}{n}\right)^2 + 2\left(\frac{3k}{n}\right)(2) + 2^2\right)$$

$$= \frac{3}{n}\sum_{k=1}^{n}\left(\frac{9k^2}{n^2} + \frac{12k}{n} + 4\right) = \frac{3}{n}\sum_{k=1}^{n}\frac{9}{n^2}k^2 + \frac{3}{n}\sum_{k=1}^{n}\frac{12k}{n} + \frac{3}{n}\sum_{k=1}^{n}4$$

$$= \frac{27}{n^3}\sum_{k=1}^{n}k^2 + \frac{36}{n^2}\sum_{k=1}^{n}k + \frac{12}{n}\sum_{k=1}^{n}1$$

$$= \frac{27}{n^3}\left(\frac{n^3+\sim}{3}\right) + \frac{36}{n^2}\left(\frac{n^2+\sim}{2}\right) + \frac{12}{n}\cdot n$$

$$\xrightarrow{n\to\infty} \frac{27}{3} + \frac{36}{2} + 12 = 9 + 18 + 12 = 39 \checkmark$$

$$\int_2^5 x^2\,dx = \left.\frac{x^3}{3}\right]_2^5 = \frac{5^3}{3} - \frac{2^3}{3} = \frac{125-8}{3} = \frac{117}{3} = 39 \checkmark$$

A proof I left out of the videos:

$$\sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2$$

Pf

$$\sum_{k=1}^{1} k^3 = 1^3 \overset{?}{=} \left(\frac{1(1+1)}{2}\right)^2 = \left(\frac{2}{2}\right)^2 = 1^2 = 1 \checkmark$$

S $\exists\, k \in \mathbb{N} \ni$

$$\sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2.$$ Consider $$\sum_{k=1}^{n+1} k^3 = 1 + 2^3 + 3^3 + \dots + n^3 + (n+1)^3$$

$$= \sum_{k=1}^{n} k^3 + (n+1)^3$$

$$= \left(\frac{n(n+1)}{2}\right)^2 + (n+1)^3$$

$$= \left(\frac{n^2+n}{2}\right)^2 + n^3 + 3n^2 + 3n + 1$$

$$= \frac{n^4 + 2n^3 + n^2}{4} + \frac{4(n^3+3n^2+3n+1)}{4} =$$

$$= \frac{n^4 + 2n^3 + n^2 + 4n^3 + 12n^2 + 12n + 4}{4} = \frac{n^4 + 6n^3 + 13n^2 + 12n + 4}{4}$$ ugh! Hard to simplify!

We want this to be $\left(\frac{n(n+1)}{2}\right)^2$

Go back to this step:

$$= \left(\frac{n(n+1)}{2}\right)^2 + (n+1)^3$$

and FACTOR, rather than expanding:

$$= \frac{n^2(n+1)^2}{4} + \frac{4(n+1)^3}{4} = \frac{(n+1)^2\left[n^2 + 4(n+1)\right]}{4}$$

$$= \frac{(n+1)^2(n^2+4n+4)}{4} = \frac{(n+1)^2(n+2)^2}{4} = \left(\frac{(n+1)(n+2)}{2}\right)^2$$

$\Rightarrow$ we are done! This shows $\sum_{k=1}^{n+1} k^3 = \left(\frac{(n+1)(n+2)}{2}\right)^2$

so $P(n)$ true $\Rightarrow P(n+1)$ true. Since $P(1)$ is true,

so is $P(1+1) = P(2)$, $P(2+1) = P(3)$, ..., $P(n)$ for any $n \in \mathbb{N}$ $\blacksquare$