Only 2 midterm re-takes scheduled. I expected more. But I'm not going to worry about your grade more than you do.

Today, we do a quick tour of the resources on harryzaims.com. I'm getting questions about this or that resource, which I assumed everyone was accessing with ease.

On the next page, I compute a definite integral by the limit definition.

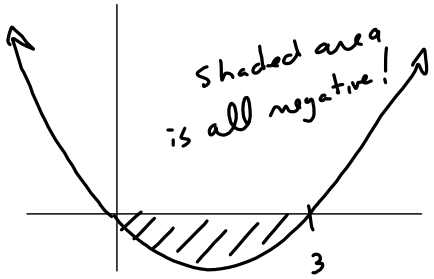
From Test 4, Fall '13:

Evaluate the definite integral by the limit definition:

$$\int_0^3 (x^2-3x)\,dx$$

This is saying "Find the 'signed' area 'under' the function x^2-3x on the interval [0,3].

Quick Pic:



$x(x-3)=0 \Rightarrow x=0,3$

$a=0,\ b=3$

$\frac{b-a}{n}=\frac{3-0}{n}=\frac{3}{n}=\Delta x$

$x_k=a+k\left(\frac{b-a}{n}\right)=0+k\left(\frac{3}{n}\right)=\frac{3k}{n}$

$a=0$ is nice!

$$\Delta x\sum_{k=1}^{n} f(x_k)=\frac{3}{n}\sum_{k=1}^{n} f\left(\frac{3k}{n}\right)$$

$$=\frac{3}{n}\sum_{k=1}^{n}\left(x_k^2-3x_k\right)=\frac{3}{n}\sum_{k=1}^{n}\left(\left(\frac{3k}{n}\right)^2-3\left(\frac{3k}{n}\right)\right)$$

$$=\frac{3}{n}\sum_{k=1}^{n}\left(\frac{9k^2}{n^2}-\frac{9k}{n}\right)=\frac{3}{n}\sum_{k=1}^{n}\frac{9}{n^2}k^2-\frac{3}{n}\sum_{k=1}^{n}\frac{9}{n}k$$

$$=\frac{3}{n}\left(\frac{9}{n^2}\right)\sum_{k=1}^{n}k^2-\frac{3}{n}\cdot\frac{9}{n}\sum_{k=1}^{n}k$$

$$=\frac{27}{n^3}\left[\frac{n(n+1)(2n+1)}{6}\right]-\frac{27}{n^2}\left[\frac{n^2+n}{2}\right]$$

$\frac{n^2}{2}$)

Better Double-Check

Cheat-Cheet Material:

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} = \frac{n^2 + \text{lower degree}}{2} \rightarrow \text{main thing.}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} = \frac{2n^3 \pm \text{lower degree}}{6} = \frac{n^3 + \text{lower degree}}{3}$$

$$\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2 = \frac{n^4 + \text{lower degree}}{4}$$

$$= \frac{27}{n^3}\left[\frac{n(n+1)(2n+1)}{6}\right] - \frac{27}{n^2}\left[\frac{n^2+n}{2}\right]$$

$$= \frac{27}{n^3}\left[\frac{n^3+n}{3}\right] - \frac{27}{n^2}\left[\frac{n^2+n}{2}\right] \xrightarrow{n\to\infty} \left(\frac{27}{n^3}\right)\left(\frac{n^3}{3}\right) - \frac{27}{n^2}\left(\frac{n^2}{2}\right)$$

$$= 9 - \frac{27}{2} = \frac{18-27}{2} = \boxed{-\frac{9}{2}}$$

Same thing, using FTC II from later on:

$$\int_0^3 (x^2-3x)\,dx = \left[\frac{x^3}{3} - 3\cdot\frac{x^2}{2}\right]_0^3 = \frac{9^3}{3} - \frac{3}{2}(3^2) = 9 - \frac{27}{2} = -\frac{9}{2}$$ ✓