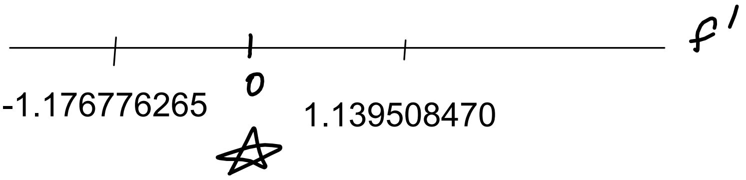
Section 3.6 #7 - I would use WolframAlpha.com. At least one has gotten through this #7 just using the notes on harryzaims.com.

Desmos will plot anything, but Wolfram Alpha will actually compute derivatives and find their zeros as well as finding the zeros of their denominators (The other half of "critical points."

Wolfram will compute the derivatives and find all their zeros and blow-ups.

"Differentiate (x^(2/3))/(x^4+x+9) works perfectly. You can write that down and then enter it and find its zeros with "Solve (your derivative) = 0. It's quite understanding, if you know the mathematical objects you seek, so you know how to ask for them.

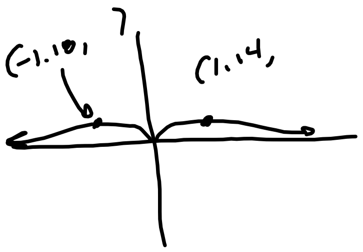


$$f'(x)=\frac{10x^4+x-18}{3x^{1/3}\left(x^4+x+9\right)^2}$$

critical: $x \approx -1.18, 0, 1.14$

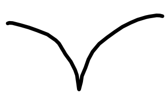
$$f''(x)=\frac{2\left(65x^8-14x^5-720x^4+2x^2-72x-81\right)}{9x^{4/3}\left(x^4+x+9\right)^3}$$

Desmos Graphing Calculator Graph is something like this:



Increasing: $(-\infty,-1.18)\cup(0,1.14)$

The above graph was what suggested to me what the sign pattern must be for the derivative. It's increasing from the far left to that first peak at -1.18. The x^(2/3) piece contributes a sort of "cusp" structure:



cusp @ $x=0$ from the $x^{\frac{2}{3}}$ piece.

3.7 #4

Consider the following problem: A box with an open top is to be constructed from a square piece of cardboard, 3 ft wide, by cutting out a square from each of the four corners and bending up the sides. Find the largest volume that such a box can have.

4. (a) Draw several diagrams to illustrate the situation, some short boxes with large bases and some tall boxes with small bases. Find the volumes of several such boxes.

(b) Draw a diagram illustrating the general situation. Let x denote the length of the side of the square being cut out. Let y denote the length of the base.

(c) Write an expression for the volume V in terms of both x and y.

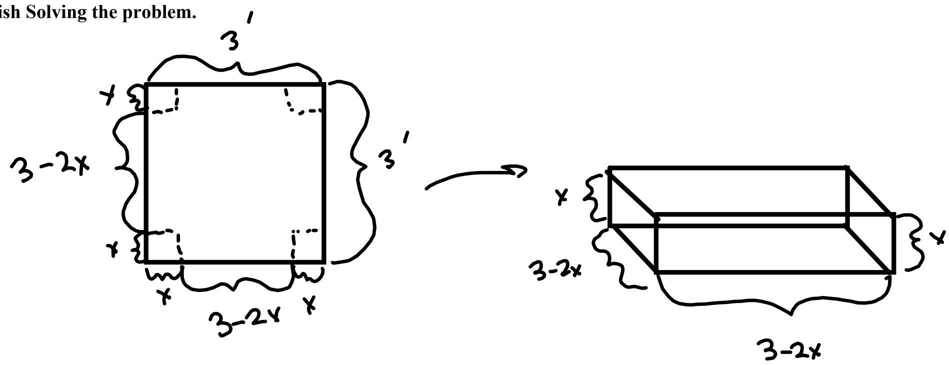
$V =$ ✗ xy^2

(d) Use the given information to write an equation that relates the variables x and y.

✗ $3 = 2x + y$

(e) Use part (d) to write the volume as a function of only x.

(f) Finish Solving the problem.



Volume = $l \cdot w \cdot h$

$V(x) = (3-2x)(3-2x)(x)$

$x(3-2x)^2 = x(2x-3)^2$

$\Rightarrow V'(x) = (2x-3)^2 + x(2(2x-3)(2))$

$= (2x-3)(2x-3+x(4))$

$= (2x-3)(6x-3) \overset{\text{SET}}{=} 0 \to$

$x = \frac{3}{2}$ OR $x = \frac{1}{2}$

$= 3(2x-3)(2x-1)$

$V'(x) = (-2)(3-2x)(x)$

$+ (-2)(3-2x)(x)$

$+ (2x-3)^2$

$= -4(3-2x)(x) + (2x-3)^2$

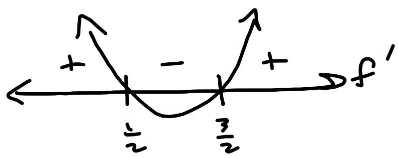
$= -4(3x-2x^2) + 4x^2 - 12x + 9$

$= -12x + 8x^2 + 4x^2 - 12x + 9$

$= 12x^2 - 24x + 9$

$= 3(4x^2 - 8x + 3)$

$= 3(2x-1)(2x-3)$



$\frac{1}{2}$ MAX $\frac{3}{2}$ MIN

$V\left(\frac{1}{2}\right) = \frac{1}{2}\left(2\left(\frac{1}{2}\right) - 3\right)^2$

$= \frac{1}{2}(-2)^2 = 2$ = MAX VOL

$= 2\ ft^3$

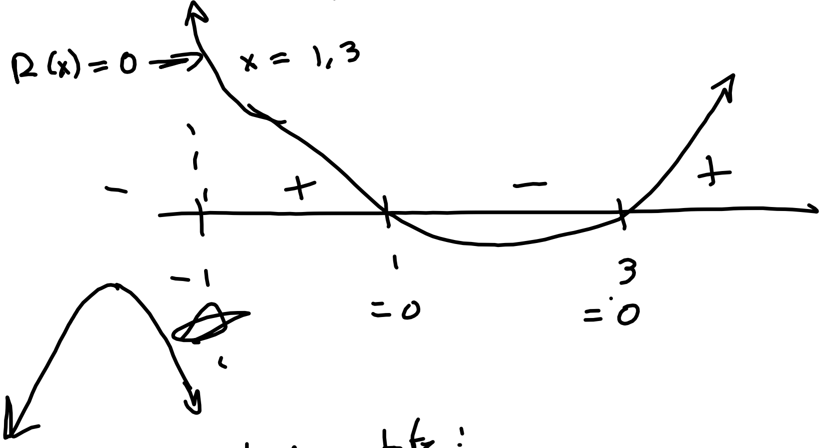
$= (3-2x)(3-2x)(x) = x(2x-3)^2$

#2 is the Rational Function on Writing Project #2. Here is the same question (different version) from Spring, 2024:

$$R(x) = \frac{x^2-4x+3}{x+1} = \frac{(x-3)(x-1)}{(x+1)}$$

$\mathcal{D} = \mathbb{R} \setminus \{-1\} \rightarrow \boxed{x=-1 \text{ is V.A.}}$

$R(x) = 0 \rightarrow x = 1, 3$



Slant Asymptote:

Divide $\quad x+1 \overline{\smash{)}\, x^2-4x+3}$

$$\begin{array}{r|rrr} -1 & 1 & -4 & 3 \\ & & -1 & 5 \\ \hline & 1 & -5 & 8 \end{array} \rightarrow x-5 \text{ r } 8$$

$$\frac{x^2-4x+3}{x+1} = x-5 + \frac{8}{x+1}$$

$\rightarrow$ Slant Asymptote