Boy, oh boy! This one was a handful! Very technical execution. Nothing hard about it, but when I got in a hurry I made small errors that would have thrown off the whole thing.

If I were doing it for points, I would be very methodical, with lots more writing than I did trying to perform it live. I got just a couple coefficients slightly off in my calculation of f' and it threw everything off.

The other thing I missed on the first pass was the fact that the zeros of the denominator of f' were of multiplicity 2, which means the sign doesn't change as the graph crosses the $x = -2$ asymptote and the $x = 3$ asymptote.

On f'', the denominator is cubed (after you do some simplifying). '3' is odd. Therefore the sign of f'' does change as we cross over the vertical asymptotes.

3.6 #2

Define $f(x)$:

$f := x \rightarrow \frac{(x^4 - x^3 - 8)}{x^2 - x - 6}$

$$f := x \mapsto \frac{x^4 - x^3 - 8}{x^2 - x - 6} \qquad \textbf{(1.1)}$$

Compute f':

$fp := \mathrm{D}(f)$

$$fp := x \mapsto \frac{4 \cdot x^3 - 3 \cdot x^2}{x^2 - x - 6} - \frac{(x^4 - x^3 - 8) \cdot (2 \cdot x - 1)}{(x^2 - x - 6)^2} \qquad \textbf{(1.2)}$$

Define f'':

$fpp := \mathrm{D}(fp)$

$$fpp := x \mapsto \frac{12 \cdot x^2 - 6 \cdot x}{x^2 - x - 6} - \frac{2 \cdot (4 \cdot x^3 - 3 \cdot x^2) \cdot (2 \cdot x - 1)}{(x^2 - x - 6)^2} + \frac{2 \cdot (x^4 - x^3 - 8) \cdot (2 \cdot x - 1)^2}{(x^2 - x - 6)^3} - \frac{2 \cdot (x^4 - x^3 - 8)}{(x^2 - x - 6)^2} \qquad \textbf{(1.3)}$$

Simplify f':

$simplify(fp(x))$

$$\frac{2x^5 - 4x^4 - 22x^3 + 18x^2 + 16x - 8}{(x^2 - x - 6)^2} \quad \textbf{(1.4)}$$

Find the (real) zeros of *f*:

$solve(f(x) = 0)$

$$2, -\frac{(46 + 3\sqrt{249})^{1/3}}{3} + \frac{5}{3(46 + 3\sqrt{249})^{1/3}} - \frac{1}{3}, \frac{(46 + 3\sqrt{249})^{1/3}}{6} - \frac{5}{6(46 + 3\sqrt{249})^{1/3}} - \frac{1}{3} + \frac{I\sqrt{3}\left(-\frac{(46 + 3\sqrt{249})^{1/3}}{3} - \frac{5}{3(46 + 3\sqrt{249})^{1/3}}\right)}{2}, \frac{(46 + 3\sqrt{249})^{1/3}}{6} - \frac{5}{6(46 + 3\sqrt{249})^{1/3}} - \frac{1}{3} - \frac{I\sqrt{3}\left(-\frac{(46 + 3\sqrt{249})^{1/3}}{3} - \frac{5}{3(46 + 3\sqrt{249})^{1/3}}\right)}{2} \quad \textbf{(1.5)}$$

What a mess! Evaluate floating-point:

$evalf(\%)$

$$2., -1.477967242, 0.2389836218 - 1.627669118\,I, 0.2389836218 + 1.627669118\,I \quad \textbf{(1.6)}$$

So *f* has 2 real zeros, not that WebAssign asked for them.

Now find the zeros of *f* ':

$solve(fp(x) = 0)$

$$RootOf(\_Z^5 - 2\_Z^4 - 11\_Z^3 + 9\_Z^2 + 8\_Z - 4, index = 1), RootOf(\_Z^5 - 2\_Z^4 - 11\_Z^3 + 9\_Z^2 + 8\_Z - 4, index = 2), RootOf(\_Z^5 - 2\_Z^4 - 11\_Z^3 + 9\_Z^2 + 8\_Z - 4, index = 3), RootOf(\_Z^5 - 2\_Z^4 - 11\_Z^3 + 9\_Z^2 + 8\_Z - 4, index = 4), RootOf(\_Z^5 - 2\_Z^4 - 11\_Z^3 + 9\_Z^2 + 8\_Z - 4, index = 5) \quad \textbf{(1.7)}$$

Another mess! Let's clean it up:

$evalf(\%)$

$$0.4110790456, 1.082024449, 4.058901063, -0.8071908715, -2.744813686 \quad \textbf{(1.8)}$$

So there are FIVE real zeros of f':
0.4110790456, 1.082024449, 4.058901063, −0.8071908715, −2.744813686.

Now, let's look at the 2nd derivative:

$fpp(x)$

$$\frac{12 x^2 - 6 x}{x^2 - x - 6} - \frac{2 \left(4 x^3 - 3 x^2\right) (2 x - 1)}{\left(x^2 - x - 6\right)^2} + \frac{2 \left(x^4 - x^3 - 8\right) (2 x - 1)^2}{\left(x^2 - x - 6\right)^3} - \frac{2 \left(x^4 - x^3 - 8\right)}{\left(x^2 - x - 6\right)^2} \quad \textbf{(1.9)}$$

Clean up f'':

$simplify(\%)$

$$\frac{2 x^6 - 6 x^5 - 30 x^4 + 82 x^3 + 348 x^2 - 168 x - 112}{\left(x^2 - x - 6\right)^3} \quad \textbf{(1.10)}$$

Find the zeros of f'':

$solve(fpp(x) = 0)$

$$RootOf(\_Z^6 - 3\,\_Z^5 - 15\,\_Z^4 + 41\,\_Z^3 + 174\,\_Z^2 - 84\,\_Z - 56, index = 1), RootOf(\_Z^6 - 3\,\_Z^5 - 15\,\_Z^4 + 41\,\_Z^3 + 174\,\_Z^2 - 84\,\_Z - 56, index = 2), RootOf(\_Z^6 - 3\,\_Z^5 - 15\,\_Z^4 + 41\,\_Z^3 + 174\,\_Z^2 - 84\,\_Z - 56, index = 3), RootOf(\_Z^6 - 3\,\_Z^5 - 15\,\_Z^4 + 41\,\_Z^3 + 174\,\_Z^2 - 84\,\_Z - 56, index = 4), RootOf(\_Z^6 - 3\,\_Z^5 - 15\,\_Z^4 + 41\,\_Z^3 + 174\,\_Z^2 - 84\,\_Z - 56, index = 5), RootOf(\_Z^6 - 3\,\_Z^5 - 15\,\_Z^4 + 41\,\_Z^3 + 174\,\_Z^2 - 84\,\_Z - 56, index = 6) \quad \textbf{(1.11)}$$

Clean up the solution by making it floating point:

$evalf(\%)$

$$0.7903183252, 4.056456160 + 1.816126406\,I, -2.757822170 + 1.284335499\,I, -0.3875863048, -2.757822170 - 1.284335499\,I, 4.056456160 - 1.816126406\,I \quad \textbf{(1.12)}$$

I see 0.7903183252, −0.3875863048 as *the* two real zeros. Don't forget the zeros of the denominator when you do your sign pattern!