Weird Scheduling on WebAssign...

We will move some of the early Chapter 3 back a bit,

and the rest of the stuff, later, forward a bit.

Done.

There'll still be adjustments to the schedule on WebAssign,

but it didn't make sense to have a 10-day hole in assignments

and then rush everything at the end.

This is more in keeping with the overview provided by the

Course Schedule.

We can *always* talk about moving things back, as needed.

You know the drill. Circle final answers. Show all work, etc.

1. (10 pts) Sketch the graph of $f(x) = 16x^3 - 24x^2 - 63x + 98$. Show all intercepts, asymptotes, extremes and inflection points.

Quick Graph suggests zeros at x = -2, 7/4...

$$-2 \,\underline{|\,16 \quad -24 \quad -63 \quad 98}$$
$$ -32 \quad 112 \quad -98$$
$$\overline{16 \quad -56 \quad 49 \quad 0}$$

$\Rightarrow (x+2)(16x^2 - 56x + 49)$

Looks Like Perfect square trinomial

$a^2 - 2ab + b^2$

$(4x)^2 - (2)(4x)(7) \quad +7^2 \quad$ yup

$= (x+2)(4 \cdot x - 7)^2$

$x = -2, \dfrac{7}{4}$

College Algebra

$(-2,0)$ $(0,98)$ $(\frac{7}{4},0)$

Now do calculus:

$f'(x) = \dfrac{48x^2 - 48x - 63}{3} \overset{SET}{=} 0$

$\Rightarrow 16x^2 - 16x - 21 = 0$
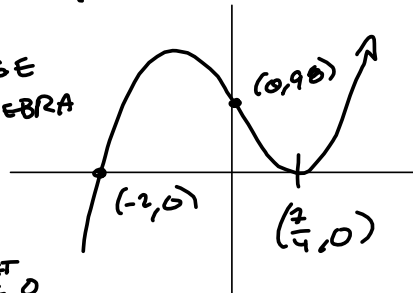
$b^2 - 4ac = 16^2 - 4(16)(21)$
$= 256 + 64(21)$
$= 256 + 1344$
$= 1600 \quad = 16 \cdot 100$
$\sqrt{1600} = 4 \cdot 10 = 40$

$\begin{array}{r} 64 \\ 21 \\ \hline 64 \\ 128 \\ \hline 1344 \end{array}$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{16 \pm 40}{2(48)} \rightarrow \frac{56}{96} = \frac{28}{48} = \frac{7}{12} \cdot \frac{14}{24} = \frac{7}{12}$$

$$\searrow \frac{-24}{96} =$$

$$16x^2 - 16x - 21$$

$$(16)(21)$$
$$= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 7$$

want a diff of -16:

$$\color{red}{x^2 - x - \frac{21}{16} = 0}$$

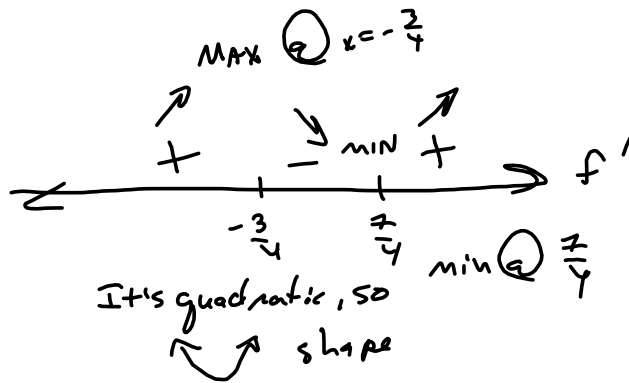$$x^2 - x + \left(\frac{1}{2}\right)^2 - \frac{1}{4} - \frac{21}{16} = 0$$

$$\left(x - \frac{1}{2}\right)^2 - \frac{4 - 21}{16} = 0$$

$$\left(x - \frac{1}{2}\right)^2 = \frac{25}{16}$$

$$x - \frac{1}{2} = \pm\sqrt{\frac{25}{16}} = \pm\frac{5}{4}$$
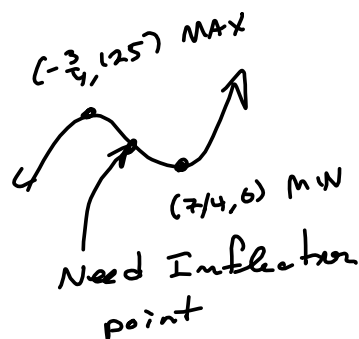
$$x = \frac{2 \pm 5}{4} \nearrow \frac{7}{4}$$
$$\searrow -\frac{3}{4}$$

MAX @ $x = -\frac{3}{4}$

$$+ \quad - \quad \text{MIN} \quad +$$

$$-\frac{3}{4} \qquad \frac{7}{4} \qquad f'$$

min @ $\frac{7}{4}$

It's quadratic, so ⤴ shape

$$f\left(-\frac{3}{4}\right) = \; ?$$

$$-\frac{3}{4} \rfloor \quad 16 \quad -24 \quad -63 \quad 98$$

$$\quad\quad\quad -12 \quad 27 \quad 27$$

$$\quad 16 \quad -36 \quad -36 \quad \boxed{125 = f\left(-\frac{3}{4}\right)}$$

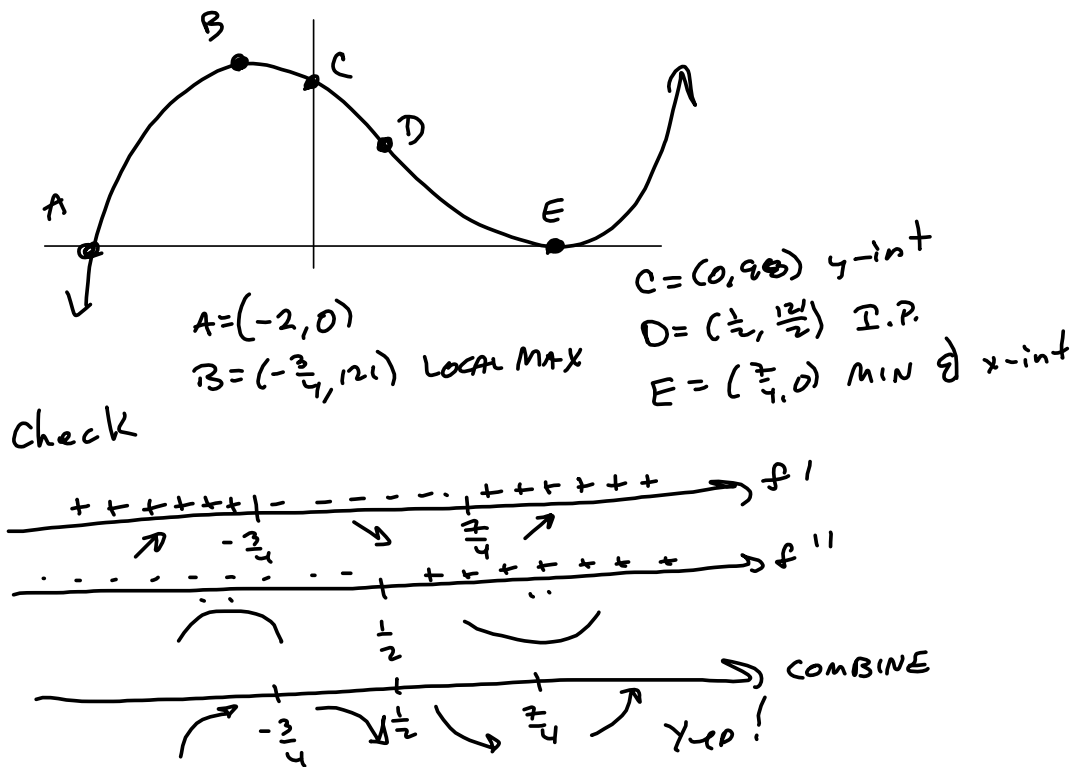$$f'(x) = 48x^2 - 48x - 63 \;\Longrightarrow$$

$$f''(x) = 96x - 48 \overset{SET}{=} 0 \;\Longrightarrow$$

$$96x = 48 \;\Longrightarrow$$

$$x = \frac{1}{2}$$

$$f\left(\tfrac{1}{2}\right) = \; ?$$

$$\begin{array}{r} 98 \\ 2 \\ \hline 176 \end{array}$$

$$\frac{1}{2} \rfloor \quad 16 \quad -24 \quad -63 \quad 98 \quad\quad 176$$

$$\quad\quad\quad\quad 8 \quad\quad 8 \quad -55 \quad\quad -55$$

$$\quad\quad\quad\quad\quad\quad\quad\quad\quad 2 \quad\quad\quad 121$$

$$16 \quad -16 \quad -55 \quad \boxed{\begin{array}{c} \tfrac{21}{2} = f\left(\tfrac{1}{2}\right) \\ I.P. \end{array}}$$

(-3/4, 125) MAX

(7/4, 6) MIN

Need Inflection point

B
C
D
A
E

$A = (-2, 0)$
$B = (-\frac{3}{4}, 121)$ LOCAL MAX

$C = (0, 98)$ y-int
$D = (\frac{1}{2}, \frac{121}{2})$ I.P.
$E = (\frac{7}{4}, 0)$ MIN & x-int

Check

$+ + + + + + | - - - - - - | + + + + + \rightarrow f'$
$\nearrow \quad -\frac{3}{4} \quad \searrow \quad \frac{7}{4} \quad \nearrow$

$\cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot | + + + + + + + \rightarrow f''$
$\frown \quad \frac{1}{2} \quad \smile$

$\rightarrow$ COMBINE

$\nearrow \quad -\frac{3}{4} \quad \searrow \frac{1}{2} \searrow \quad \frac{7}{4} \quad \nearrow$ YEP!

For graphing, we want to do the sign patterns on f' and f'' before making the graph, but we know what a cubic polynomial should look like, and it's just a matter of finding zeros of f' and f'' to tell us.

For messier functions involving trig functions, we don't have the same level of intuition.