Fermat's Theorem

If $f(c)$ is local max & $f'(c)$ exists, then $f'(c) = 0$

Pf

§ $f(c)$ is a local max & h is "small"

Then $f(c+h) \le f(c)$

$\Rightarrow f(c+h) - f(c) \le 0$ Assume $h > 0 \Rightarrow$

$$\Rightarrow \frac{f(c+h) - f(c)}{h} \le 0$$

$$f'(c) = \lim_{h \to 0^+} \frac{f(c+h) - f(c)}{h} \le 0$$

Assume $h < 0$.

Then $f(c+h) - f(c) \le 0$, again, but now $h < 0$.

$$\frac{f(c+h) - f(c)}{h} \ge 0$$

$$\Rightarrow f'(c) = \lim_{h \to 0^-} \frac{f(c+h) - f(c)}{h} \ge 0$$

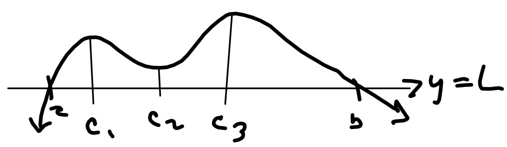
$0 \le f'(c) \le 0 \Rightarrow f'(c) = 0$

Rolle's:

Given: f cnt^s on $[a,b]$,

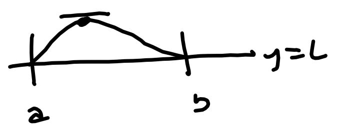
f difb^l on (a,b), and $f(a)=f(b)=L$

$\Rightarrow \exists\, c \in (a,b) \ni f'(c)=0$



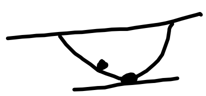
Pf If $f(x)=L$ $\forall$ $x\in(a,b)$, then we're done. $f'(c)=0$ $\forall$ $c\in(a,b)$

If $f(x)>L$ anywhere, then f has an abs max in $[a,b]$, and it's not @ a or b, so, by Fermat, $f'(c)=0$ somewhere in (a,b)



If $f(x)<L$ anywhere, then f has an abs min* in $[a,b]$ and it's not @ a or b

By Fermat, $f'(c)=0$



* at some $x=c$, i.e., $f(c)$ is a min

Find the absolute maximum and absolute minimum values of f on the given interval.

$f(x) = 2x^3 - 3x^2 - 36x + 9, \quad [-3, 4]$

$$\begin{array}{r|rrrr} -3 & 2 & -3 & -36 & 9 \\ & & -6 & 27 & 27 \\ \hline & 2 & -9 & -9 & \boxed{36 = f(-3)} \end{array}$$

Neither (After checking f')

$$\begin{array}{r|rrrr} 4 & 2 & -3 & -36 & 9 \\ & & 8 & 20 & -64 \\ \hline & 2 & 5 & -16 & \boxed{-55 = f(4)} \end{array}$$

Neither (After Checking f')

Now $f'(x) = 6x^2 - 6x - 36 = 6(x^2 - x - 6) = 6(x-3)(x+2) = 0 \Rightarrow$

$x = -2, 3$

$$\begin{array}{r|rrrr} -2 & 2 & -3 & -36 & 9 \\ & & -4 & 14 & 44 \\ \hline & 2 & -7 & -22 & \boxed{53 = f(-2)} \end{array}$$

After Comparing

Abs & Local max

$$\begin{array}{r|rrrr} 3 & 2 & -3 & -36 & 9 \\ & & 6 & 9 & -81 \\ \hline & 2 & 3 & -27 & \boxed{-72 = f(3)} \end{array}$$

After Comparing

Abs & Local Min