Use a linear approximation (tangent-line approximation) to approximate $\tan(28^\circ)$

Let $L(x)$ = tangent line $L(x) = f'(x_1)(x - x_1) + f(x_1)$

We know 28° is close to 30° & we know $\tan(30^\circ) =$

Convert to radians. $(30^\circ)\left(\frac{\pi}{180^\circ}\right) = \boxed{\frac{\pi}{6} = x_1}$

$28^\circ = (28^\circ)\left(\frac{\pi}{180^\circ}\right) = (7)\left(\frac{\pi}{45}\right) = \frac{7\pi}{45}$

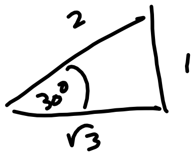
$L(x) = f'\left(\frac{\pi}{6}\right)\left(x - \frac{\pi}{6}\right) + f\left(\frac{\pi}{6}\right)$

$= f'(x_1)(x - x_1) + f(x_1)$

$f'(x) = \sec^2(x)$

$f'(x_1) = f'\left(\frac{\pi}{6}\right) = \sec^2\left(\frac{\pi}{6}\right)$

$= \left(\frac{2}{\sqrt{3}}\right)^2 = \frac{4}{3} = f'(x_1)$



$f(x_1) = \tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}}$

$L(x) = \frac{4}{3}\left(x - \frac{\pi}{6}\right) + \frac{1}{\sqrt{3}}$

$L\left(\frac{7\pi}{45}\right) = \frac{4}{3}\left(\frac{7\pi}{45} - \frac{\pi}{6}\right) + \frac{1}{\sqrt{3}}$

$3\,|\,45$, $3\,|\,15$, 5 $\quad$ $2\,|\,6$, 3

$\text{LCD} = 3 \cdot 3 \cdot 2 \cdot 5 = 90$

$.45 \times 3 = 135$

$= \frac{4}{3}\left(\frac{2 \cdot 7\pi}{90} - \frac{15\pi}{90}\right) + \frac{1}{\sqrt{3}}$

$= \frac{4}{3}\left(-\frac{\pi}{90}\right) + \frac{1}{\sqrt{3}}$

$= \boxed{-\frac{2\pi}{135} + \frac{1}{\sqrt{3}}} \approx \tan(28^\circ) \approx$ 0.5308081559 (Tan. Line Approx.)

$\tan(28^\circ) \approx$ 0.5317094320 (Calculator)

$L(28^\circ) \approx$ 0.5308081559

Needs to be in radians!

$\frac{1}{\sqrt{3}}$ sucks by hand

1.732050808 $\overline{)\,1.000000000}$

$\frac{\sqrt{3}}{3}$ is better:

$$\begin{array}{r} .57 \\ 3\,\overline{)\,1.732050808} \\ -1.5 \\ \hline .232050808 \end{array}$$

Finishing up 2.9

What's an error in measurement do to the error in the calculation? We have a way to estimate that, using differentials.

The measurement of the radius of a sphere is 10 cm +/- 0.2 cm

Use differentials to approximate the error in the calculation of the volume.

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dr} = 4\pi r^2$

$dV = 4\pi r^2 dr$

$dr = 0.2 = \text{error}$

$r = 10 \text{ cm}$

$\Delta V \approx dV = 4\pi(10^2)(.2) = 4\pi(100)(.2) = 4\pi(20) = \boxed{80\pi \text{ cm}^3}$

That looks big!

Check: $V(10.2) - V(10)$

$\frac{4}{3}\pi(10.2)^3 - \frac{4}{3}\pi(10^3)$

$V(10.2) - V(10) \approx 256.387471 \text{ cm}^3$

$evalf(80 \cdot \text{Pi}) \approx 251.3274123 \text{ cm}^3$

RELATIVE ERROR:

$\frac{\Delta V}{V} \approx \frac{dV}{V} = \frac{80\pi}{\frac{4}{3}\pi(10)^3} = 3/50 = \frac{6}{100} = \boxed{.06 \text{ relative error}}$

PERCENT ERROR = (RELATIVE ERROR)(100 %) = $\boxed{6\% \text{ PERCENT ERROR}}$

Verify the given linear approximation at $a = 0$. Then determine the values of x for which the linear approximation is accurate to within 0.1. (Enter your answer using interval notation. Round your answers to three decimal places.)

$$\frac{1}{(1+2x)^4} \approx 1 - 8x$$

$f(x) = (2x+1)^{-4}$

$f'(x) = -4(2x+1)^{-5}(2)$

$f'(0) = -4(1)^{-5}(2) = -8$

$f(0) = 1$

$L(x) = f'(x_1)(x - x_1) + f(x_1)$

$= -8(x-0) + 1 \quad = -8x + 1$ ✓

want

$\left| \frac{1}{(2x+1)^4} - (-8x+1) \right| < .1$

$\left| \frac{1 - (-8x+1)(2x+1)^4}{(2x+1)^4} \right| < .1$

$-.1 <$ ~ $< .1$

$.1(2x+1)^4 < 1 - (-8x+1)(2x+1)^4 < .1(2x+1)^4$

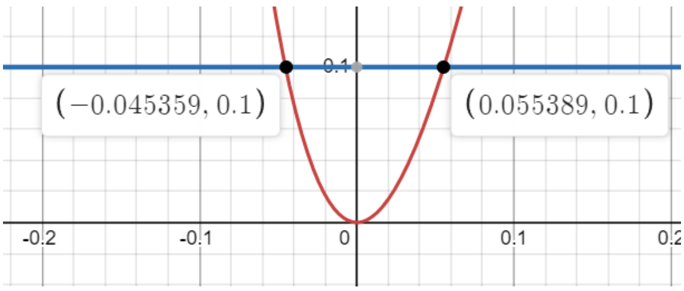
$.1(2x+1)^4 < 1 + 8x(2x+1)^4 - (2x+1)^4 < .1(2x+1)^4$

Looks tough to do, analytically, but Deesmos has an answer.

Graph f(x) and y = 0.1 and y = - 0.1. Find where they intersect.

(-0.045359, 0.1)

(0.055389, 0.1)



$x \in (-.045, .055)$ will keep $|f(x) - L(x)| < .1$