2.7 #10

$\frac{dC}{dt} = aC - bCW$

$= C(a - bW)$

$= 0$ when $C = 0$ or $W = \frac{a}{b}$

$\frac{dW}{dt} = -cW + dCW$

$= -W(c - dC)$

$= 0$ when $W = 0$ or $C = \frac{c}{d}$

Let $\boxed{W = \frac{a}{b},\ C = \frac{c}{d} \quad \text{STABLE (Equilibrium)}}$

Then $\frac{dC}{dt} = \frac{c}{d}\left(a - b\left(\frac{a}{b}\right)\right) = 0$

$\frac{dW}{dt} = -\frac{a}{b}\left(c - d\left(\frac{c}{d}\right)\right) = 0$

(c) $a = .06,\ b = .001,\ c = .06,\ d = .0001$

stable solutions?

$(C, W) = (0, 0)$

$(C, W) = \left(\frac{c}{d}, \frac{a}{b}\right) = \left(\frac{.06}{.0001}, \frac{.06}{.001}\right) = \left(\frac{6 \times 10^{-2}}{1 \times 10^{-4}}, \frac{6 \times 10^{-2}}{1 \times 10^{-3}}\right)$

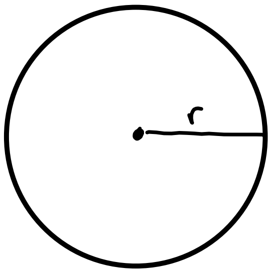
$= (6 \times 10^2, 6 \times 10^1) = (600, 60)$

§2.8 Related Rates

The area of the ripples from a pebble dropped in the pond is related to the rate at which the radius increases (how fast the wave front is moving).

Say the rate of radius increase is 40 ft/sec

$\frac{dr}{dt} = 40 \frac{ft}{sec}$



How fast is the area inside increasing?

$A = \pi r^2$ Now $r = r(t)$ use chain rule

$A = \pi r^2 \longrightarrow$

$\frac{dA}{dt} = \pi\left(2r \cdot \frac{dr}{dt}\right)$ by chain rule

How fast is area changing when $r = 10\,ft$?

$\frac{dA}{dt} = \pi(2(10)(40))$

$= \pi(2(10\,ft)(40\,\frac{ft}{s}))$

$= \boxed{800\pi\,\frac{ft^2}{s}}$

Midterm Test:

October 14th or 15th

From around 8 am - 6 pm (start test).

In a room in Horizon Hall, to be announced.

Give yourself 2 hours.

1 page, 2-sided cheat sheet allowed.

Scientific calculator required.

Graphing calculator forbidden.

I will announce the room in Horizon Hall and confirm the time window. But I think it's from around 8 am to around 6 pm for start time.

Need to take your test somewhere else? We can arrange a proctored test in your area, but you need to speak up.

Midterm Review

Chapter 2 Test Portion

Questions from Test 2, Spring '15:

1. Let $f(x) = \sqrt{x+1}$.
   a. (5 pts) Find an equation of the tangent line to f at the point $(3, 2)$.
   b. (5 pts) Sketch a graph showing f and the tangent line to f at the point $(3, 2)$.

$f(x) = (x+1)^{\frac{1}{2}}$

$f'(x) = \frac{1}{2}(x+1)^{-\frac{1}{2}}(1)$

$= \frac{1}{2\sqrt{x+1}}$

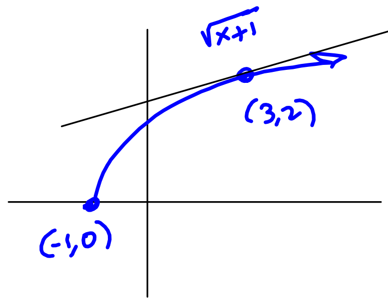
$f'(3) = \frac{1}{2\sqrt{3+1}} = \frac{1}{2\sqrt{4}} = \frac{1}{2 \cdot 2}$

$= \frac{1}{4} = m_{tan} = f'(3)$

$f'(x_1)(x-x_1)+f(x_1)$

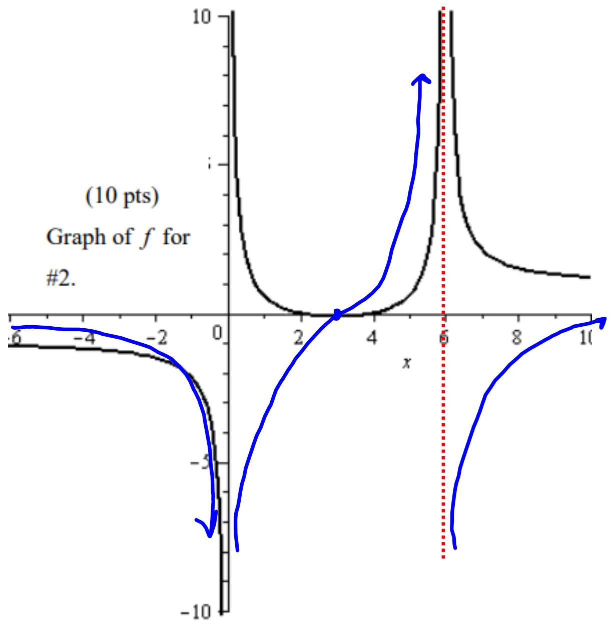
$y = m(x-x_1)+y_1$

$$\boxed{y = \frac{1}{4}(x-3)+2}$$



$y = \frac{1}{4}(x-3)+2$ = tangent line.

2. (10 pts) The graph of a function f is given on the right. On the same set of axes, sketch a graph of f'.



(10 pts) Graph of f for #2.

f'

f

3. (5 pts each) Differentiate the following with respect to the independent variable.

a. $f(x) = x^5 \cancel{- 6x^{\frac{7}{3}} + 6\sqrt[3]{x^7}} + 4x^{\frac{2}{5}} - \frac{3}{2}x^{-\frac{2}{3}}$

$\sqrt[3]{x^2} = x^{\frac{2}{3}}$

$\Rightarrow f'(x) = 5x^4 \cancel{- \frac{42}{3}x^{\frac{4}{3}}} + \frac{8}{5}x^{-\frac{3}{5}} + 1x^{-\frac{5}{3}}$

$= 5x^4 + \frac{8}{5}x^{-\frac{3}{5}} + x^{-5/3}$

b. $h(\omega) = (\omega^2 + 3\omega + 13)(\omega^3 - 7\omega^2) \Rightarrow$

$(fg)' = f'g + fg'$

$\boxed{h'(\omega) = (2\omega + 3)(\omega^3 - 7\omega^2) + (\omega^2 + 3\omega + 13)(3\omega^2 - 14\omega)}$

c. $H(t) = \dfrac{t^2 + 3t}{t^3 + 6t - 11} \Rightarrow$

$\left(\frac{f}{g}\right)' = \dfrac{f'g - fg'}{g^2}$

$\boxed{H'(t) = \dfrac{(2t+3)(t^3+6t-11) - (t^2+3t)(3t^2+6)}{t^3+6t}}$

d. $g(x) = (x^2 + 3x + 13)^3 (x^3 - 7x^2)^{-5} \Rightarrow$

$\boxed{g'(t) = 3(x^2+3x+13)^2(2x+3)(x^3-7x^2)^{-5} + (x^3+3x+13)^3(-5(x^3-7x^2)^{-6}(3x^2-14x)}$

e. $r(x) = \dfrac{(x^2 + 3x)^3}{(x^3 - 7x^2)^5}$

$\boxed{r'(x) = \dfrac{3(x^2+3x)^2(2x+3)(x^3-7x^2)^5 - (x^2+3x)^3(5(x^3-7x^2)^4(3x^2-14x)}{(x^3-7x^2)^{10}}}$

f. $Q(t) = \dfrac{\sin(t^2 - 3t)}{\cos(5t)} \Rightarrow$

$\boxed{Q'(t) = \dfrac{\cos(t^2-3t)(2t-3)(\cos(5t)) - \sin(t^2-3t)(-\cos(5t)(5))}{\cos^2(5t)}}$

g. $R(x) = \dfrac{\csc^3(5x)}{\tan(\pi x)} \Rightarrow$

$\boxed{R'(x) = \dfrac{(3\csc^2(5x))(-\csc(5x)\cot(5x))(5)(\tan(\pi x)) - \csc^3(5x)(\sec^2(\pi x)(\pi))}{\tan^2(\pi x)}}$

4. (10 pts) Show that $f(x)=x^3-6x^2+15x-7$ has no tangent line with a slope of $m=-2$.

$f'(x)=3x^2-12x+15$ SET -2

$3x^2-12x+17=0$

$a=3,\ b=-12,\ c=17 \Rightarrow$

$b^2-4ac = 12^2-4(3)(17)$

$= 12(12-17)$

$= 12(-5)$

$= -60 < 0 \Rightarrow$ No real solns $\Rightarrow$

No $x \ni f'(x)=-2$ ∎

5. Consider the relation $y\sin(2x)=x\cos(2y)$.

a. (5 pts) Use implicit differentiation to find $y'=\frac{dy}{dx}$.

Chain Rule

$\Rightarrow y'\sin(2x)+y(\cos(2x)(2)) = \cos(2y)+x(-\sin(2y))(2y')$

$\Rightarrow \sin(2x)y'+2\cos(2x)y = \cos(2y)-2x\sin(2y)y'$

$\Rightarrow (\sin(2x)+2x\sin(2y))y' = \cos(2y)-2x\sin(2y)$

$\Rightarrow \boxed{y'=\frac{\cos(2y)-2x\sin(2y)}{\sin(2x)+2x\sin(2y)}}$

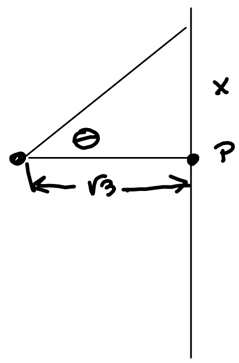
b. (5 pts) Find an equation of the tangent line to the curve at the point $\left(\frac{\pi}{2},\frac{\pi}{4}\right)$.

$$y'\Big|_{(x,y)=\left(\frac{\pi}{2},\frac{\pi}{4}\right)} = \frac{\cos(2(\frac{\pi}{4}))-2(\frac{\pi}{2})\sin(2(\frac{\pi}{4}))}{\sin(2(\frac{\pi}{2}))+2(\frac{\pi}{2})\sin(2(\frac{\pi}{4}))} = \frac{\cos(\frac{\pi}{2})-\pi\sin(\frac{\pi}{2})}{\sin(\pi)+\pi(\sin(\frac{\pi}{2}))} = \frac{-\pi(1)}{\pi(1)} = -1$$

$\boxed{m=-1}$

$\boxed{y=-1\left(x-\frac{\pi}{2}\right)+\frac{\pi}{4}}$

6. (10 pts) A lighthouse is located on a small island *exactly* $\sqrt{3}$ km from the nearest point P on a straight shoreline. The light makes 5 revolutions per minute. How fast is the beam of light moving along the shoreline when it is 1 km away from P?



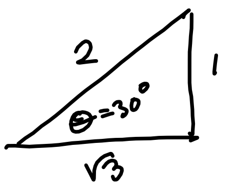
We obtain angular velocity from rpm's.

$$\frac{d\theta}{dt} = \left(\frac{5 \text{ revs}}{\text{min}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right) = \frac{10\pi}{\text{min}} \text{ radians}$$

$$\frac{x}{\sqrt{3}} = \tan\theta$$

$$x = \sqrt{3}\tan\theta \quad (\text{in Km})$$

$$\frac{dx}{dt} = \sqrt{3}\sec^2(\theta)\frac{d\theta}{dt}$$



$30° = \frac{\pi}{4}$

$\arctan\left(\frac{1}{\sqrt{3}}\right) =$

$\arcsin\left(\frac{1}{2}\right)$

$$\left.\frac{dx}{dt}\right|_{x=1} = \sqrt{3}\sec^2\left(\frac{\pi}{3}\right)(10\pi)$$

$$= \sqrt{3}\left(\frac{2}{\sqrt{3}}\right)^2(10\pi)$$

$$\doteq \sqrt{3}\left(\frac{4}{3}\right)10\pi = \frac{40\sqrt{3}}{3}\pi \ \frac{\text{Km}}{\text{min}}$$

We're not quite ready for #7. It's about differentials, and I'd like more than 10 minutes to present the theory and an example.