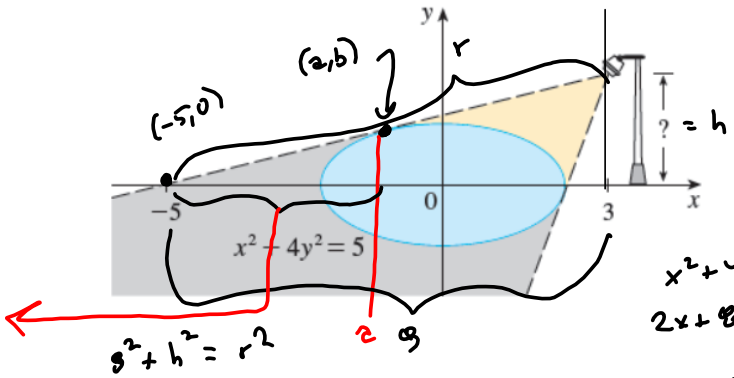
Writing Project #0 Final Draft - Due 10/6.

Section 2.6

17. [-/1 Points] DETAILS SCALC8 2.6.062.

The figure shows a lamp located three units to the right of the y-axis and a shadow created by the elliptical region $x^2 + 4y^2 \le 5$. If the point $(-5, 0)$ is on the edge of the shadow, how far above the x-axis is the lamp located?



$s^2 + h^2 = r^2$

$x^2+4y^2 = 5$: boundary

$2x + 8yy' = 0$

$y' = -\frac{2x}{8y} = -\frac{x}{4y}$

we don't know x, y.

(3,h), (-5,0) on the line

$m = \frac{0-h}{-5-3} = \frac{h}{8}$

$y = m(x-x_1)+y_1$

$= \frac{h}{8}(x-(-5))+0$

$= \frac{h}{8}x + \frac{5h}{8}$

(2,b), slope is $-\frac{2}{4b}$

$\frac{-2}{4b} = \frac{5h}{8}$

$m_{lam} = \frac{b}{|2-(-5)|} = \frac{b}{2+5}$

$|2+5|$

Distance from 2 to -5

$\frac{b}{2+5} = -\frac{2}{4b} \Rightarrow 4b^2 = -2^2 - 5a$

$b^2 = \frac{-a^2-5a}{4}$

$b = \pm\sqrt{\frac{-a^2-5a}{4}}$

$= -\frac{\sqrt{-a^2-5a}}{2}$

Also $a^2 + 4b^2 = 5$

$a = -\sqrt{5-4b^2}$

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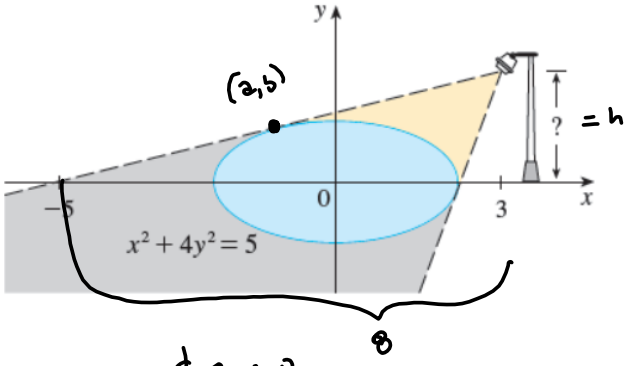


$\frac{h}{\theta} = m_{tan}$

$x^2+4y^2=5$

$2x+8yy'=0$

$y' = -\frac{x}{4y} = m_{tan}$

Also $m_{tan} = \frac{b}{|a+5|}$

$= \frac{b}{\text{dist from } a \text{ to } 5} = \frac{b}{a+5}$

since $a > -5$ & $a < 0$

$a+5 > 0$

$|a+5| = a+5$

$\Rightarrow \frac{b}{a+5} = -\frac{a}{4b}$

$\Rightarrow 4b^2 = -a^2 - 5a$

$a^2+4b^2 = -5a$.

But $a^2+4b^2=5$ on the boundary of ellipse $\Rightarrow$

$a = -1$, since $-5a = 5$

$\frac{b}{a+5} = -\frac{a}{4b}$

$\frac{b}{-1+5} = -\frac{(-1)}{4b}$

$4b^2 = 4$

$b^2 = \pm 1$

$b = -1$ No, idiot. $b > 0 \Rightarrow b = 1$

$m = \frac{h}{\theta} = \frac{b}{a+5} = \frac{-1}{-1+5} = \frac{-1}{4} = \frac{1}{4}$

$\Rightarrow \frac{h}{\theta} = \frac{1}{4} \Rightarrow h = 2$

At last!

Test 2 Part I covers 2.1 - 2.5

Take that this week.

Test 2 Part II next week.

Due Date is the 14th, but you want to beat that deadline, so you can prep for the midterm.

9. (5 pts) Prove that the equation $\sin\left(\frac{\pi}{3}x\right) = x - 1$ has a root in the interval $(0,3)$, but *do not solve*!

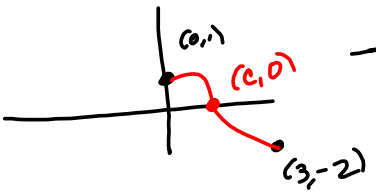
$\sin(\frac{\pi}{3}x) = x-1$ if and only if

$f(x) = \sin(\frac{\pi}{3}x) - x + 1 = 0$

$f(x) = 0$ means $\sin(\frac{\pi}{3}x) = x-1$ has a solution.

$f(0) = \sin(0) - 0 + 1 = 1 = f(0)$

$f(3) = \sin(\frac{\pi}{3}\cdot 3) - 3 + 1 = \sin(\pi) - 2 = 0 - 2 = -2 < 0$ $= f(3)$



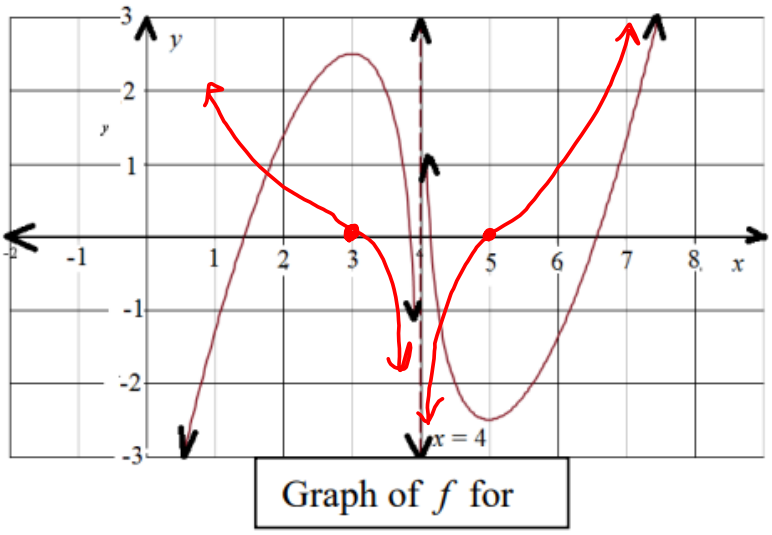
$\longrightarrow$ $f(x) = 0$ somewhere between $x = 0$ & $x = 3$

f is continuous,

$f(0) = 1 > -2 = f(3)$ $\longrightarrow$

There is a $c \in (0,3)$ such that $f(c) = 0$ by IVP

Intermediate Value Theorem.



Graph of f for

$f'(x)$ in Red

Midterm Prep: We're all the way up to #s 5 & 6 on the Fall '17 Test 2, and that's the extent of Midterm content.

Also look at the other semesters' test 1's and 2's.

Newton's Apple.

Let $s(t)$ = height of falling body from

s_0 = initial height, with

v_0 = " velocity (up or down) &

g = acceleration due to gravity.

Then $s(t) = -\frac{1}{2}gt^2 + v_0 t + s_0 \Rightarrow$

$s'(t) = -gt + v_0$ = velocity $= v(t)$

$\Rightarrow s''(t) = v'(t) = -g$

On Earth, $g = 9.8 \frac{m}{sec^2} = 9.8 \frac{m}{s^2}$

On the Moon, $g \approx .83$