Questions on 2.4? 2.5?

https://harryzaims.com/

$x^3\cos(x)$

$f = x^3 \quad f' = 3x^2$

$g = \cos(x) \quad g' = -\sin(x)$

$(fg)' = f'g + fg'$

$$\frac{d}{dx}\left[\left(\sin\left(x^3\cos(x)\right)\right)^5\right]$$

$$= 5\left(\sin\left(x^3\cos(x)\right)\right)^4 \cos\left(x^3\cos(x)\right)\left[\underbrace{3x^2}_{f'}\underbrace{\cos(x)}_{g} + \underbrace{x^3}_{f}\underbrace{\sin(x)}_{g'}\right]$$

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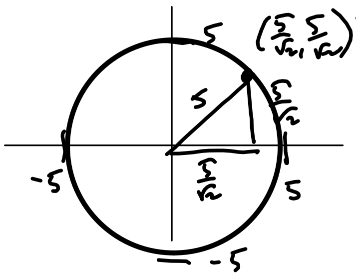
§2.6 Implicit Differentiation.

Good for equations like a circle:

$x^2+y^2=25$



Not a function, but locally, it sort of is a function $y=f(x)$

Assume y "sort of" is a function of x.

Then $\frac{d}{dx}[y] = \frac{d}{dx}[y(x)]$

$\frac{d}{dx}$ so $\frac{d}{dx}[y^2] = \frac{d}{dx}[(y(x))^2]$

$= 2y(x)\cdot\frac{dy}{dx}$

$= 2yy'$

$$\frac{d}{dx}\left[7y^5\right] = 35y^4 \cdot y'$$

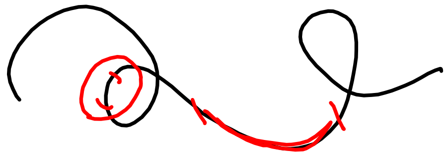
$x^2+y^2 = 25$

$y^2 = 25-x^2$

$y = \pm\sqrt{25-x^2}$

$y = \sqrt{25-x^2}$ Top $\frac{1}{2}$

$y = -\sqrt{25-x^2}$ Bottom $\frac{1}{2}$



2.6 Says:

$\frac{d}{dx}\left[x^2+y^2 = 25\right]$ Assume $y = y(x)$

$2x + 2yy' = 0$

$2yy' = -2x$

$\boxed{y' = -\frac{x}{y}}$ Gives slope @ any pt. (x,y) on the curve.

Find an equation of the tangent line to the circle at the point $\left(\frac{5}{\sqrt{2}}, \frac{5}{\sqrt{2}}\right)$.

$$y' = -\frac{x}{y} = -\frac{\frac{5}{\sqrt{2}}}{\frac{5}{\sqrt{2}}} = -1 \qquad \text{we have } (x_1, y_1) = $$

$$y = m(x - x_1) + y_1 = \boxed{-1\left(x - \frac{5}{\sqrt{2}}\right) + \frac{5}{\sqrt{2}} = y = L(x) = \text{tangent line}}$$

old way:

$$y = (25 - x^2)^{\frac{1}{2}}$$

$$y' = \frac{1}{\cancel{2}}(25 - x^2)^{-\frac{1}{2}}(-\cancel{2}x)$$

$$y'\Big|_{x = \frac{5}{\sqrt{2}}} = \left(\sqrt{25 - \left(\frac{5}{\sqrt{2}}\right)^2}\right)\left(-\frac{5}{\sqrt{2}}\right) = \left(25 - \frac{25}{2}\right)^{-\frac{1}{2}}\left(-\frac{5}{\sqrt{2}}\right)$$

$$= \left(\frac{25}{2}\right)^{-\frac{1}{2}}\left(-\frac{5}{\sqrt{2}}\right)$$

$$= \left(\frac{2}{25}\right)^{\frac{1}{2}}\left(-\frac{5}{\sqrt{2}}\right) = \frac{2^{\frac{1}{2}}}{25^{\frac{1}{2}}}\left(-\frac{5}{\sqrt{2}}\right)$$

$$= \frac{\sqrt{2}}{5} \cdot \left(-\frac{5}{\sqrt{2}}\right) = \boxed{-1 = m}$$

$$(x_1, y_1) = \left(\frac{5}{\sqrt{2}}, \frac{5}{\sqrt{2}}\right)$$

$$y = m(x - x_1) + y_1 = \boxed{-1\left(x - \frac{5}{\sqrt{2}}\right) + \frac{5}{\sqrt{2}} = y} \quad \text{Tan. Line.}$$

Try: $x^2y^3 + \sin(x)\cos(y) = 0$

Try solving for y! Can't.

Find eq'n of tangent line to the curve @ $(0,0)$

$2xy^3 + x^2(3y^2y') + \cos(x)\cos(y) + \sin(x)(-\sin(y))y' = 0$

Solve for y':

~~$(3x^2y^2 + \sin(x)\sin(y))y' = c$~~

$(3x^2y^2 - \sin(x)\sin(y))y' = -2xy^3 - \cos(x)\cos(y)$

$$y' = \frac{-2xy^3 - \cos(x)\cos(y)}{3x^2y^2 - \sin(x)\sin(y)}$$

But $y' \nexists$ @ $(x,y) = (0,0)$

Can't finish.

Might speculate that y is vertical,

so that $x=0$ would be the tangent line!

11. [–/4 Points] DETAILS **SCALC8 2.6.034.**

(a) The curve with equation $y^2 = x^3 + 3x^2$ is called the **Tschirnhausen cubic**. Find an equation of the tangent line to this curve at the point (1, 2).

$y =$ $\frac{9}{4}x - \frac{1}{4}$

(b) At what points does this curve have horizontal tangents?

(x, y) = ($-2, -2$) (smaller y-value)

(x, y) = ($-2, 2$) (larger y-value)

$2yy' = 3x^2 + 6x \Rightarrow$

$y' = \frac{3x^2+6x}{2y}$

$(x_1, y_1) = (1, 2) \Rightarrow$

$y'\Big|_{(x,y)=(1,2)} = \frac{3(1)^2+6(1)}{2(2)} = \frac{9}{4} = m$

$\boxed{y = \frac{9}{4}(x-1)+2}$

$= m(x - x_1) + y_1$

Horizontal Tangents when $y' = 0$

$\Rightarrow y' = \frac{3x^2+6x}{2y} = 0 \Rightarrow$

$3x^2 + 6x = 0 \Rightarrow$

$3x(x+2) = 0 \Rightarrow$

$\boxed{x = 0, x = -2}$

Plug in to find y: $y' \nexists$ @ (0,0)!

$y^2 = x^3 + 3x^2$

$x = 0 \Rightarrow y = 0 \rightsquigarrow (0,0)$

$x = -2 \Rightarrow y^2 = -8 + 12 = 4 \rightsquigarrow$ ~~(-2, 4)~~

$y^2 = 4 \Rightarrow y = \pm 2 \Rightarrow \boxed{(-2, 2),\ (-2, -2)}$

only two points