Chain Rule SPECIAL **The General Power Rule**

If you grok the Chain Rule, then you already know the General Power Rule, so it's kind of an extra thing to remember that just wastes space in your brain.

When $h(x) = f(g(x))$ & $f(u) = u^n = (g(x))^n$

$$\frac{d}{dx}[h(x)] = \frac{d}{dx}[f(g(x))] = \frac{d}{dx}[f(u)] = \frac{d}{dx}[u^n]$$

$$= n u^{n-1} \frac{du}{dx}$$

$$\frac{d}{dx}\left[(x^7 + \sin(x))^5\right] = 5(x^7 + \sin(x))^4 \cdot (7x^6 + \cos(x))$$

since $\frac{d}{dx}[\sin(x)] = \cos(x)$.

Find the limit.

$$\lim_{\theta\to 0}\frac{\cos(4\theta)-1}{\sin(7\theta)} = \lim_{\theta\to 0}\left(\frac{\cos(4\theta)-1}{1}\cdot\left(\frac{1}{\sin(7\theta)}\right)\right)$$

$$= \lim_{\theta\to 0}\left(\frac{\cos(4\theta)-1}{\theta}\cdot\frac{\theta}{\sin(7\theta)}\right)$$

$$= \lim_{\theta\to 0}\left(\left(\frac{\cos(4\theta)-1}{4\theta}\cdot 4\right)\left(\frac{7\theta}{\sin 7\theta}\cdot\frac{1}{7}\right)\right)$$

$$= \lim_{\theta\to 0}\left(\frac{4\cos(4\theta-1)}{4\theta}\right)\lim_{\theta\to 0}\left(\frac{7\theta}{7\sin(7\theta)}\right)$$

$$= 4\lim_{\theta\to 0}\left(\frac{\cos(4\theta)-1}{4\theta}\right)\left(\frac{1}{7}\lim_{\theta\to 0}\frac{7\theta}{\sin(7\theta)}\right)$$

$$= 4\cdot 0\cdot\frac{1}{7}\cdot 1$$

$\lim_{\theta\to 0}\frac{\sin(3\theta)}{\theta} = 3$, but it's NOT because you factored the "3" out of the $\sin(3\theta)$

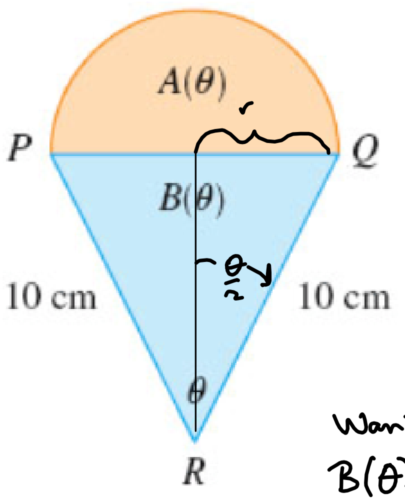
$$= 4\lim_{\theta\to 0}\left(\frac{\cos(4\theta)-1}{4\theta}\right)\left(\frac{1}{7}\lim_{\theta\to 0}\frac{7\theta}{\sin(7\theta)}\right)$$

$$= 4\lim_{u\to 0}\left(\frac{\cos(u)-1}{u}\right)\left(\frac{1}{7}\lim_{v\to 0}\left(\frac{v}{\sin(v)}\right)\right),$$ where $u=4\theta, v=7\theta$

& Note that $\theta\to 0$ if and only if $4\theta\to 0$ & $7\theta\to 0$

A semicircle with diameter PQ sits on an isosceles triangle PQR to form a region shaped like a two-dimensional ice-cream cone, as shown in the figure. If $A(\theta)$ is the area of the semicircle and $B(\theta)$ is the area of the triangle, find

$$\lim_{\theta \to 0^+} \frac{A(\theta)}{B(\theta)}.$$



See 2.4 Notes on harryzaims.com by clicking here. There's a way to simplify this problem that I missed, live, and then went and looked up in the 2.4 Notes, where it was plain as day.

want,

$A(\theta)$

$A(r) = \frac{1}{2}\pi r^2$

$\frac{r}{10} = \sin\left(\frac{\theta}{2}\right)$

$r = 10\sin\left(\frac{\theta}{2}\right)$

$\therefore A(r) = A(r(\theta)) = \frac{1}{2}\pi\left(10\sin\left(\frac{\theta}{2}\right)\right)^2$

$= 50\pi\sin^2\left(\frac{\theta}{2}\right)$

Want

$B(\theta)$:

$B = \frac{1}{2}bh = \frac{1}{2}br = \frac{1}{2}b\sin\left(\frac{\theta}{2}\right)$

$\cancel{r^2 + h^2 = 10^2}$

$r^2 + b^2 = 10^2$

$\sqrt{b^2} = \sqrt{100 - r^2}$

$b = \sqrt{100 - r^2}$ (b/c $b > 0$)