Find constants A and B such that the function $y = A\sin(x) + B\cos(x)$ satisfies the differential equation $y'' + y' - 5y = \sin(x)$.

A = [ ]

B = [ ]

22. [–/2 Points] DETAILS MY NOTES SCALC8 2.4.053.

Linear 2nd-order Ordinary Differential Equation

y, y', y'' to 1st power

y'' in it

$y^{(2)} \ldots$

$y = A\sin(x) + B\cos(x)$

$y' = A\cos(x) - B\sin(x)$

$y'' = -A\sin(x) - B\cos(x)$

$y'' + y' + 5y = (-A\sin(x) - B\cos(x)) + (A\cos(x) - B\sin(x))$ $\cancel{+5(A\sin(x) + B\cos(x))}$

$\cancel{= (-A - B + 5A)\sin(x) + (-B + A + 5B)\cos(x)}$

$\cancel{= (-4A - B)\sin(x) + (A + 4B)\cos(x) = \sin(x)}$

$-A\sin(x) - B\cos(x) - A\cos(x) - B\sin(x) - 5(A\sin(x) + B\cos(x))$

$= -A\sin(x) - B\cos(x) - A\cos(x) - B\sin(x) - 5A\sin(x) - 5B\cos(x)$

$= \sin(x)(-A - B - 5A) + \cos(x)(-B - A - 5B)$

$= \sin(x)(-6A - B) + \cos(x)(-6B - A) \overset{\text{WANT}}{=} \sin(x)$

$-6A - B = 1 \quad (1)$

$-A - 6B = 0 \quad (2)$

M1 SUBSTITUTION

$-A = 6B$

$A = -6B$, By (2).

& so

$-6A - 6B =$

$-6(-6B) - 6B$

$= 36B - 6B$

$= 30B = 1$ (see (1))

$B = \frac{1}{30}$

& $A = -6B = -6\left(\frac{1}{30}\right) = -\frac{1}{5}$

$\boxed{A = -\frac{1}{5},\ B = \frac{1}{30}}$

M2 ELIMINATION:

$-6A - 6B = 1$

$-A - 6B = 0$

E1 $\quad A + 6B = 0$

E2 $\quad -6A - 6B = 1$

E1+E2: $\quad -5A = 1$

$\boxed{A = -\frac{1}{5}}$

$A + 6B = -\frac{1}{5} + 6B = 0$

$-\frac{1}{5} = -6B$

$\boxed{B = \frac{1}{30}}$

using Maple to evaluate the LHS:

$6A\sin(x) + 6B\cos(x) + A\cos(x) - B\sin(x) = \sin(x)$

(1) $6A - B = 1$ (sin(x) coefficient)

(2) $A + 6B = 0$ (cos(x) ")

(2) $\boxed{A = -6B} \Rightarrow$

(1) $6(-6B) - B = 1$

$\Rightarrow -36B - B = -37B = 1$

$\Rightarrow \boxed{B = -\frac{1}{37}}$

$\Rightarrow A = -6\left(\frac{-1}{37}\right) = \boxed{\frac{6}{37} = A}$

$(-6)\,(A + 6B = 0)$

$6A - B = 1$

$-6(1)+(2)$ $\quad -6A - 36B = 0$

$6A - B = 1$

$\boxed{-37B = 1}$

$\boxed{B = -\frac{1}{37}}$ CORRECT (victor)

$A = ?$

$A + 6\left(-\frac{1}{37}\right) = 0$

$\boxed{A = \frac{6}{37}}$

$$-6\,A\sin(x) - 6\,B\cos(x) + A\cos(x) - B\sin(x)$$

$-6A - B = 1$

$A - 6B = 0$

$A = 6B$

Recall - Composition of functions $y = (f \circ u)(x) = f(u(x)) = f$ composed with u of x. A function of a function.

Example: $y = h(x) = (3x-2)^2$

Let $u(x) = 3x-2$ and $f(u) = u^2$

Then $h(x) = f(u(x)) = u(x)^2 = (3x-2)^2$

We have a trick for taking the derivatives of such composite functions that is very useful:

The Chain Rule If g is differentiable at x and f is differentiable at $g(x)$, then the composite function $F = f \circ g$ defined by $F(x) = f(g(x))$ is differentiable at x and F' is given by the product

$$F'(x) = f'(g(x)) \cdot g'(x)$$

In Leibniz notation, if $y = f(u)$ and $u = g(x)$ are both differentiable functions, then

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

By this, $h'(x) = \left(f(u(x))\right)' = \frac{dh}{dx} = \frac{df}{du}\cdot\frac{du}{dx}$

$\frac{df}{du} = \frac{d}{du}[u^2] = 2u$, $\frac{du}{dx} = 3 \Rightarrow$

for comparison:

$\frac{dh}{dx} = \frac{dh}{du}\cdot\frac{du}{dx} = 2u\cdot 3 = 6u = \boxed{6(3x-2)} = 18x-12$

Old way

$h(x) = (3x-2)^2 = (3x)^2 - 2(3x)(2) + 2^2 = 9x^2 - 12x + 4$

$\Rightarrow h'(x) = 18x - 12$

$(a+b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

$y = (7x-5)^{15}$

$\Rightarrow \boxed{y' = \underbrace{15(7x-5)^{14}}_{\frac{df}{du}}\underbrace{(7)}_{\frac{du}{dx}}}$

$\frac{d}{dx}[\sin^2(x)]$

$= \frac{d}{dx}[(\sin(x))^2]$

$= \boxed{2\sin(x)\cdot\cos(x)}$

Proof

Let $y = f(u(x))$, where u is dif'ble at x and f is dif'ble at $u(x)$.

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{u(x+\Delta x) - u(x)}{\Delta x}$$

Define $\Delta u = u(x+\Delta x) - u(x)$

& $\varepsilon_1 = \varepsilon_1(x) = \frac{\Delta u}{\Delta x} - u'(x)$ $(\Delta x \neq 0)$

$\approx \frac{\Delta u}{\Delta x}$ when $\Delta x \doteq$ small."

If u is dif'ble, then ε_1 is small when Δx is small

$\Rightarrow$ $\boxed{\varepsilon_1 \Delta x + u'(x)\Delta x = \Delta u}$ $= (\varepsilon_1 + u'(x))\Delta x$

$\Rightarrow \varepsilon_1 + u'(x) = \frac{\Delta u}{\Delta x}$

$\Rightarrow (u'(x) + \varepsilon_1)\Delta x = \Delta u$

Define $\Delta y = f(u+\Delta u) - f(u)$

& $\varepsilon_2 = \frac{\Delta y}{\Delta u} - f'(u)$

$\Rightarrow$ $\boxed{\varepsilon_2 \Delta u + f'(u)\Delta u = \Delta y}$ $= (\varepsilon_2 + f'(u))\Delta u$, i.e

$\Delta y = (f'(u) + \varepsilon_2)\Delta u$

$\Delta y = (f'(u) + \varepsilon_2)(u'(x) + \varepsilon_1)\Delta x$

$\Rightarrow \frac{\Delta y}{\Delta x} = (f'(u) + \varepsilon_2)(u'(x) + \varepsilon_1) \xrightarrow{\Delta x \to 0}$ $\boxed{f'(u)u'(x) = \frac{dy}{dx}}$

Now, $\Delta x \to 0$ implies $\varepsilon_1 \to 0$ & $\varepsilon_2 \to 0$
(u dif'ble @ x) (f dif'ble @ u)

$\therefore \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx} = h'(x) = f'(u(x))u'(x)$ ∎

$$\frac{d}{dx}\left[\left(x^3+5x^2-17x+11\right)^7\right]$$

$$\frac{d\left(x^3+5x^2-17x+11\right)^7}{d\left(x^3+5x^2-17x+11\right)}\cdot\frac{d}{dx}\left(x^3+5x^2-17x+11\right)$$

$$=7\left(x^3+5x^2-17x+11\right)^6\left(3x^2+10x-17\right)$$

$$y=\left(\underbrace{x^3+\cos(x)}\right)^3\left(x^2-1\right)^4$$

$$y'=\underbrace{3\left(x^3+\cos(x)\right)^2\left(3x^2-\sin(x)\right)}_{f'}\underbrace{\left(x^2-1\right)^4}_{g}+\underbrace{\left(x^3+\cos(x)\right)^3}_{f}\underbrace{\left(4\left(x^2-1\right)^3(2x)\right)}_{g'}$$