**We're ready for WP#1 #s 1 - 3, and the Bonus on the limit of a quadratic.**

Crystal growth furnaces are used in research to determine how best to manufacture crystals used in electric components. For proper growth of a crystal, the temperature must be controlled accurately by adjusting the input power. Suppose the relationship is given by

$$T(w) = 0.1w^2 + 2.153w + 20,$$

**1.7 #**

where $T$ is the temperature in degrees Celsius and $w$ is the power input in watts.

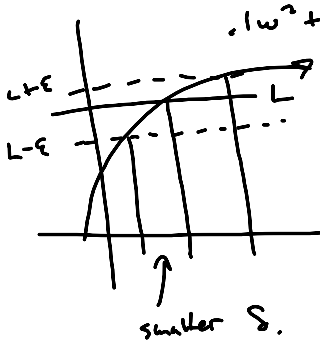
(a) How much power is needed to maintain the temperature at 202°C? (Round your answer to two decimal places.)

Soln: $T(w) = 202 = L$

Peter's got $L = 196$ & $\varepsilon =$

$.1w^2 + 2.153w + 20 = 196$

$100w^2 + 2153w + 20000 = 196000$



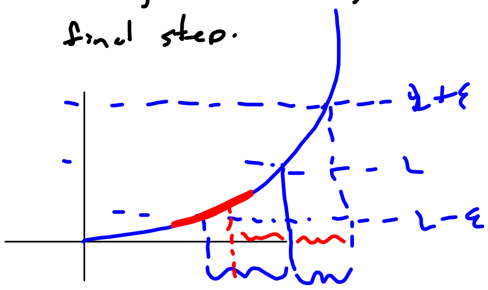

$a = 1000$

$b = 2153$

$c = 20$

etc.

Save your rounding until the final step.



Recall Tuesday:

Making a disk of $1000\,cm^2 \pm 5\,cm^2$. What's $\varepsilon$?

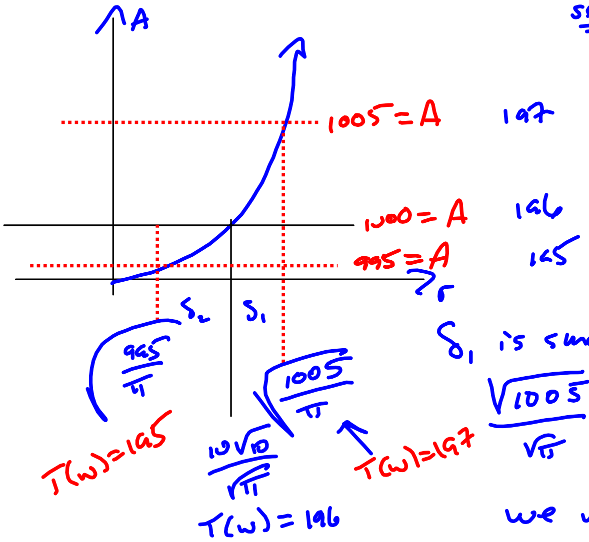
Find $\delta$.

$A = \pi r^2 \overset{SET}{=} 1000 \implies r = \dfrac{10\sqrt{10}}{\sqrt{\pi}}$

$\overset{SET}{=} 1000 + 5 = 1000 + \varepsilon = 1005$

$\implies r = \dfrac{\sqrt{1005}}{\sqrt{\pi}}$

$\overset{SET}{=} 1000 - 5 = 1000 - \varepsilon = 995 \implies r = \dfrac{\sqrt{995}}{\sqrt{\pi}}$




$\delta_1$ is smaller:

$\dfrac{\sqrt{1005}}{\sqrt{\pi}} - \dfrac{10\sqrt{10}}{\sqrt{\pi}} = \delta_2$ is the one we want, b/c $\delta_2 < \delta_1 = \dfrac{10\sqrt{10}}{\sqrt{\pi}} - \dfrac{\sqrt{995}}{\sqrt{\pi}}$

**Recall the Factor Theorem?**

If $x=c$ is a zero of a polynomial $P(x)$, then $P(x)$ has a factor of $x-c$.

Claim: $\lim_{x\to 2}(x^2+5x+6)=20$

Proof: Let $\varepsilon>0$ be given. Define $\delta=\min\{1,\frac{\varepsilon}{10}\}$. Then $0<|x-2|<\delta$ implies

$|f(x)-L|=|x^2+5x+6-20|$

$=|x+7||x-2|<10\delta\le 10\left(\frac{\varepsilon}{10}\right)=\varepsilon$

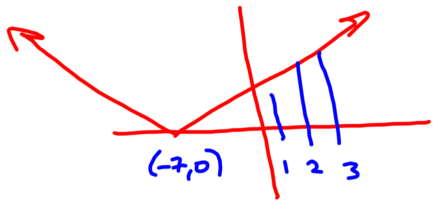
Scratch $|f(x)-L|$

$=|x^2+5x+6-20|$

$=|x^2+5x-14|$

$$\boxed{=|x+7|\underbrace{|x-2|}_{<\delta} \quad \text{WANT} <\varepsilon}$$

Need a bound on $|x+7|$




Assume $\delta\le 1$

what's the max of $|x+7|$ between $x=1$ & $x=3$?

$y=10$ is the max.

So, if $\delta\le 1 \longrightarrow |x+7|<10$.

$|x+7||x-2|<10|x-2|$

$<10\delta\le\varepsilon \Rightarrow \delta\le\frac{\varepsilon}{10}$

Now, we need to remember we assumed $\delta\le 1$.

So Define $\delta=\min\{1,\frac{\varepsilon}{10}\}$

$\lim_{x\to 2}(x^2-3x-7) = -9$

This is saying

$x^2-3x-7+9 = x^2-3x+2$ has a factor of $(x-2)$

$\lim_{x\to 1}(x^3-5x^2+2x+1) = -1$

Scratch:

$x-1$ should be a factor of $x^3-5x^2+2x+1-(-1)$
$= x^3-5x^2+2x+2 = g(x)$

$$\begin{array}{r|rrrr} 1 & 1 & -5 & 2 & 2 \\ & & 1 & -4 & -2 \\ \hline & 1 & -4 & -2 & 0 \end{array}$$ Sweet!

$\longrightarrow g(x) = (x-1)(x^2-4x-2)$

Proof

want $|f(x)-L| = |x^3-5x^2+2x+1-(-1)|$

$= |x^3-5x^2+2x+2| = |x-1|\underbrace{|x^2-4x-2|}$ WANT $< \varepsilon$.

Analyze how big this can get on $1-1=0 < x < 1+1=2$

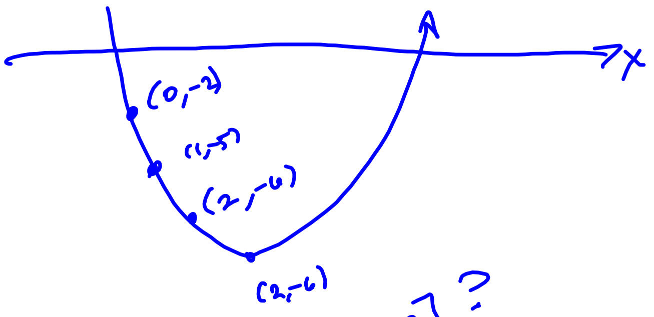
Let $\delta \le 1$. Then

$|x-1| < \delta$

& $|x^2-4x-2|$ needs a bound on $[0,2]$

$x^2-4x+2^2-2^2-2$

$= (x-2)^2-6$




What's the maximum of $|x^2-4x-2|$ on $[0,2]$? $y=6$ is max, so $\longrightarrow$

$|x-1||x^2-4x-2| < |x^2-4x-2|\delta$

$< 6$. So Define $\delta = \min\{1, \frac{\varepsilon}{6}\}$

Practice Test #1

$$\frac{x^2-x}{x-1} = \frac{x(x-1)}{x-1} = x$$

$(x \neq 1)$

$x$ is not $\frac{x^2-x}{x-1}$. NOT QUITE.

**Practice Test has same time control. Same 2 guesses as Test 1.**

**So to use it for training, make sure you look at ALL the questions and take a crack at as many as possible.**

**Practice Test does not affect your grade, except insofar as it helps you train.**

**One-Page, 2-sided cheat sheet is permitted on Test 1.**

**Scientific Calculator required. Graphing Calculator Forbidden.**

**No class on Monday, unless you have questions. Tuesday start Chapter 2.**

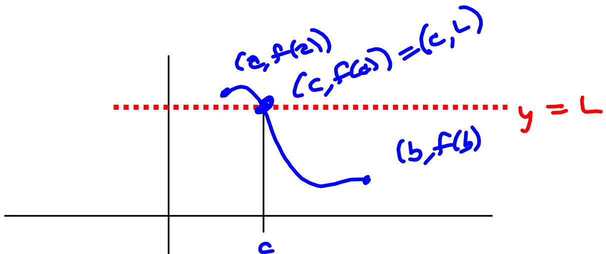
**A function *f* is a rule that assigns to each *x* in a set called the *domain* a unique *y* in a set called the *range.***

***Domain* = the set of all *x* such that *f*(*x*) is a real number.**

***Range* = the set of all *y* such that *y* = *f*(*x*) for some *x* in the Domain.**

**Intermediate Value Theorem:**

**If a chicken crosses the road, it must cross the center line.**




If f is continuous (cnt^s), with $a < b$ and $f(a) > f(b)$

and $f(a) > L > f(b)$ (i.e. L is between $f(a)$ & $f(b)$)

then there is a c in $(a,b)$ such that $f(c) = L$

**The Big Application for the test is**

Use the Intermediate Value Theorem to show that there is a root of the given equation in the specified interval.

$x^4 + x - 4 = 0, \quad (1, 2)$

Doesn't mean find it

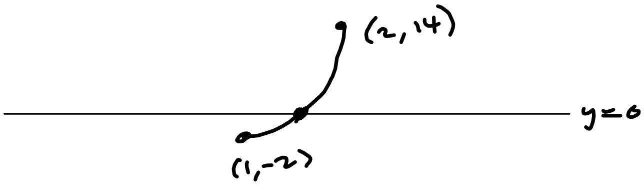
$f(x) = x^4 + x - 4$ is cnt$^s$ $\forall x \in (-\infty, \infty)$

$\forall x \in \mathbb{R}$

So it's cnt$^s$ on $(1,2)$

$f(1) = 1^4 + 1 - 4 = -2 < 0$

$f(2) = 2^4 + 2 - 4 = 14 > 0$



How close to $-1$ do we have to take $x$ so that

$$\frac{1}{(x+1)^4} > 10,000$$

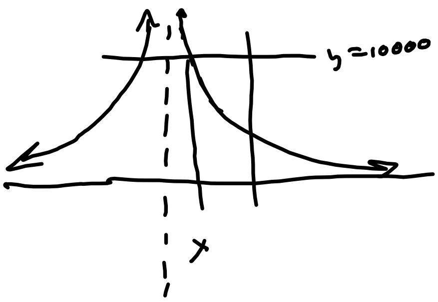
is satisfied? (Give the largest possible value.)

$|x-(-1)| <$ ☐ ✖ .1

$$\frac{1}{1000} > (x+1)^4$$

$$\sqrt[4]{\frac{1}{1000}} > |x+1| = |x-(-1)|$$

$$.1 = \frac{1}{10} = \frac{\sqrt[4]{1}}{\sqrt[4]{10000}} = \frac{1}{\sqrt[4]{10^4}} > \underbrace{|x-(-1)|}_{\text{How far from } -1 \ x \text{ is.}}$$




$\frac{1}{(x+1)^4}$ is a $\frac{1}{x^{2n}}$ shape

Solve $\frac{1}{(x+1)^4} > 1000$

$$\lim_{x\to 1} \frac{3x+9}{4} = 1$$ what's $\delta$?

$$\delta = \frac{\varepsilon}{\text{growth rate}}$$

$$\frac{3x+9}{4} = \frac{3x}{4} + \frac{9}{4} = \frac{3}{4}x + \frac{9}{4}$$

$$m = \frac{3}{4}$$

$$\delta = \frac{\varepsilon}{\frac{3}{4}}$$

$\lim_{x\to 0} x^4\cos\left(\frac{2}{x}\right) = 0$ $x^4$ is a "damping" function.

$-1 \le \cos\left(\frac{2}{x}\right) \le 1$

$-x^4 \le x^4\cos\left(\frac{2}{x}\right) \le x^4$

$-x^4 \xrightarrow{x \to 0} 0$, forces $x^4\cos\left(\frac{2}{x}\right) \xrightarrow{x \to 0} 0$, $x^4 \xrightarrow{x \to 0} 0$

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Recall: $\lim_{x\to 0} x\sin\left(\frac{\pi}{x}\right) = 0$

Proof: Assume $x > 0$

$\lim_{x\to 0^+} f(x)$ :

$-1 \le \sin\left(\frac{\pi}{x}\right) \le 1$

$-x \le x\sin\left(\frac{\pi}{x}\right) \le x$

$-x \xrightarrow{x \to 0} 0$, $\therefore x\sin\left(\frac{\pi}{x}\right) \xrightarrow{x \to 0^+} 0$, $x \xrightarrow{x \to 0^+} 0$

$\therefore =$ "therefore"

I didn't show it from the left

Assume $x < 0$

$-1 \le \sin\left(\frac{\pi}{x}\right) \le 1$

$-x \ge x\sin\left(\frac{\pi}{x}\right) \ge x$

The "$\le$" becomes "$\ge$" because $x$ is negative.

Re-write

$x \le x\sin\left(\frac{\pi}{x}\right) \le -x$

$x \xrightarrow{x \to 0} 0$, $x\sin\left(\frac{\pi}{x}\right) \xrightarrow{x \to 0^-} 0$, $-x \xrightarrow{x \to 0^-}$

$0 = \lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} f(x) \implies \lim_{x\to 0} f(x) = 0$