We're ready for WP#1 #s 1 - 3, and the Bonus on the limit of a quadratic.

Crystal growth furnaces are used in research to determine how best to manufacture crystals used in electric components. For proper growth of a crystal, the temperature must be controlled accurately by adjusting the input power. Suppose the relationship is given by

$$T(w) = 0.1w^2 + 2.153w + 20,$$

where T is the temperature in degrees Celsius and w is the power input in watts.

(a) How much power is needed to maintain the temperature at 202°C? (Round your answer to two decimal places.)

Soln: $T(w) = 202 = L$

$.1w^2 + 2.153w + 20 = 196$

Peter's got

$L = 196$

& $\varepsilon =$

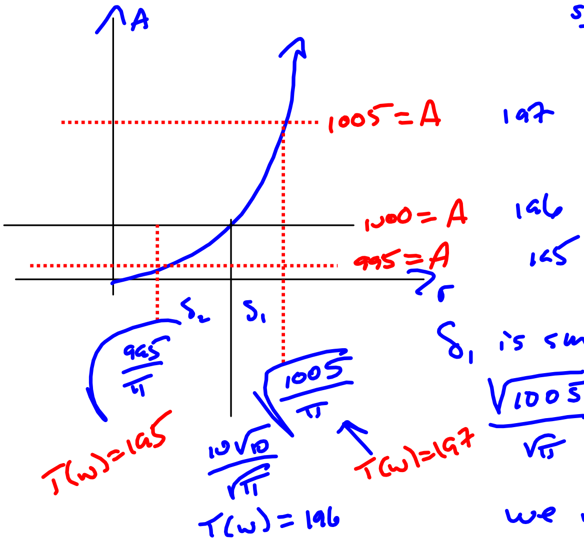
Recall Tuesday:

Making a disk of $1000\,cm^2 \pm 5\,cm^2$. What's ε?

Find δ.

$A = \pi r^2 \overset{SET}{=} 1000 \implies r = \frac{10\sqrt{10}}{\sqrt{\pi}}$

$\overset{SET}{=} 1000 + 5 = 1000 + \varepsilon = 1005 \implies r = \frac{\sqrt{1005}}{\sqrt{\pi}}$



$\overset{SET}{=} 1000 - 5 = 1000 - \varepsilon = 995 \implies r = \frac{\sqrt{995}}{\sqrt{\pi}}$

δ_1 is smaller $\therefore$ $\frac{\sqrt{1005}}{\sqrt{\pi}} - \frac{10\sqrt{10}}{\sqrt{\pi}} = \delta_2$ is the one we want, b/c $\delta_2 < \delta_1 = \frac{10\sqrt{10}}{\sqrt{\pi}} - \frac{\sqrt{995}}{\sqrt{\pi}}$

Recall the Factor Theorem?

If $x=c$ is a zero of a polynomial $P(x)$, then $P(x)$ has a factor of $x-c$.

Claim: $\lim_{x\to 2}(x^2+5x+6)=20$

Proof: Let $\varepsilon>0$ be given. Define $\delta=\min\{1, \frac{\varepsilon}{10}\}$. Then $0<|x-2|<\delta$ implies

$|f(x)-L| = |x^2+5x+6-20|$

$= |x+7||x-2| < 10\delta \le 10\left(\frac{\varepsilon}{10}\right)=\varepsilon$ ■

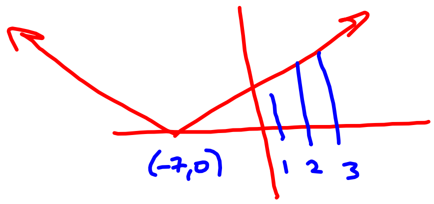
Scratch $|f(x)-L|$

$= |x^2+5x+6-20|$

$= |x^2+5x-14|$

$$\boxed{= |x+7|\underbrace{|x-2|}_{<\delta} \quad \text{WANT} < \varepsilon}$$

Need a bound on $|x+7|$



Assume $\delta \le 1$

what's the max of $|x+7|$ between $x=1$ & $x=3$?

$y=10$ is the max.

So, if $\delta \le 1 \rightarrow$

$|x+7| < 10.$

$|x+7||x-2| < 10|x-2|$

$< 10\delta \le \varepsilon \Rightarrow \delta \le \frac{\varepsilon}{10}$

Now, we need to remember we assumed $\delta \le 1$.

So Define $\delta = \min\{1, \frac{\varepsilon}{10}\}$

Evaluate the limits if they exist.

$$\lim_{x\to 3}\frac{x^2-9}{x^3-27}=\lim_{x\to 3}\frac{(x-3)(x+3)}{(x-3)(x^2+3x+9)}=\lim_{x\to 3}\frac{x+3}{x^2+3x+9}=\frac{3+3}{3^2+3^2+3^2}=\frac{6}{27}=$$

$$x^3-3^3=(x-3)(x^2+3x+3^2)$$

$$\frac{x^2-9}{x^3-27}=\frac{(x-3)(x+3)}{(x-3)(x^2+3x+9)}=\frac{x+3}{x^3+3x+9}\xrightarrow{x\to 3}\frac{3+3}{3^2+3^2+3^2}=\frac{6}{27}=\boxed{\frac{2}{9}}$$

$(x\neq 3)$

$$\lim_{x\to -3}\frac{2x^2+x-15}{x^2+5x+6}$$

$$2x^2+6x-5x-15=2x(x+3)-5(x+3)=(x+3)(2x-5)$$

$$\frac{2x^2+x-15}{x^2+5x+6}=\frac{(x+3)(2x-5)}{(x+3)(x+2)}=\frac{2x-5}{x+2}\xrightarrow{x\to -3}\frac{-6-5}{-3+2}=\frac{-11}{-1}=\boxed{11}$$

$x\neq -3$

The only way for a limit to exist when the denominator is approaching zero is if the numerator also approaches zero.

$$\lim_{x\to -3}\frac{2x^2-x-15}{x^2+5x+6}=\lim_{x\to -3}\frac{(2x+5)(x-3)}{(x+2)(x+3)}\quad \nexists$$

Note: Numerator $\xrightarrow{x\to -3}$ Not zero

Denominator $\xrightarrow{x\to -3}$ zero

$$\lim_{x\to 5}(2x-7)=3 \qquad \left(\delta=\frac{\varepsilon}{2}\right)$$

want $|f(x)-L|$

$=|2x-7-3|=|2x-10|$

$=2|x-5|<\varepsilon \Longrightarrow$

$|x-5|<\frac{\varepsilon}{2}\equiv\delta$

OR $2|x-5|<2\delta\equiv\varepsilon$

$\Longrightarrow \delta=\frac{\varepsilon}{2}$

Proof: Let $\varepsilon>0$. Define $\delta=\frac{\varepsilon}{2}$.

Then $0<|x-5|<\delta \Longrightarrow$

$|f(x)-L|=|2x-7-3|=|2x-10|$

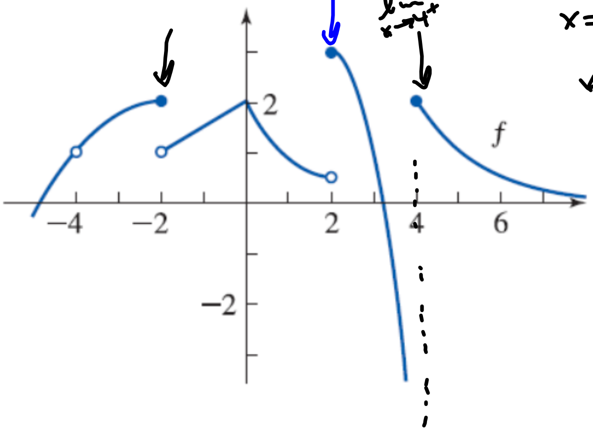
$=2|x-5|<2\delta=2\left(\frac{\varepsilon}{2}\right)=\varepsilon$ ■

Section 1.8

Use the graph to determine the *x*-values at which *f* is discontinuous. For each *x*-value, determine whether *f* is continuous from the right, from the left, or neither. (Enter your answers from smallest to largest.)

$\lim_{x\to 2^+} f(x) = f(2)$

$\lim_{x\to 4^+} f(x) = f(4)$



$x=-4$ Neither

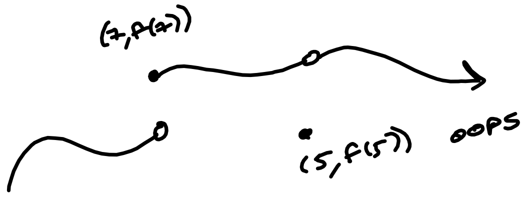
$x=-2$ Left $\lim_{x\to -2^-} f(x) = 2 = f(-2)$

$\lim_{x\to -2^+} f(x) = 1 \neq f(-2)=2$

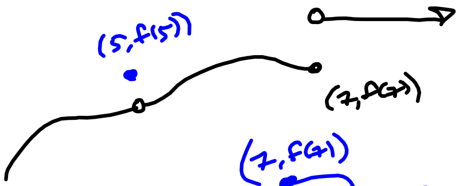
$x=2$ Right

$x=4$ Right

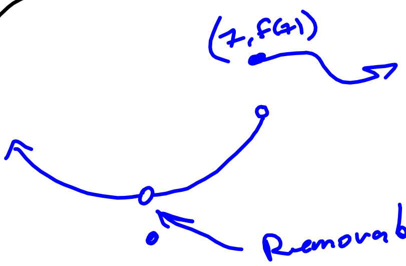
Jump Discontinuity at x = 7, Removable Discontinuity at x = 5.



oops! $5<7$!



is legit.



Removable by defining $g(x)=f(x)$ if $x\neq 5$ and $g(x) = \lim_{x\to 5} f(x)$ @ $x=5$

$$g(x)=\begin{cases} f(x) & \text{if } x\neq 5 \\ \lim_{x\to 5} f(x) & \text{if } x=5 \end{cases}$$

$f(x)= x\sin\left(\frac{\pi}{x}\right)$ has a removable discontinuity @ $x=0$

$$g(x)=\begin{cases} f(x) & \text{if } x\neq 0 \\ 0 & \text{if } x=0 \end{cases}$$ is same sort of thing.