2. [–/6.6 Points] DETAILS SCALC9 1.4.003. PRACTICE ANOTH

Re-do this w/ graphing calculator.

The point $P(6, -2)$ lies on the curve $y = \dfrac{2}{5 - x}$.

(a) If Q is the point $\left(x, \dfrac{2}{5 - x}\right)$, find the slope of the secant line PQ (correct to six decimal places) for the following values of x.

$f(x) = Y1 = 2/(5-x)$

want $m_{PQ} = \dfrac{f(x) - f(6)}{x - 6}$

$Y_2 = (Y1(x) - Y1(6))/(x-6)$

(i) 5.9

$m_{PQ} =$ ✗ 2.222222

(ii) 5.99

$m_{PQ} =$ ✗ 2.020202

VARS, YVARS, Y2

(iii) 5.999

$m_{PQ} =$ ✗ 2.002002

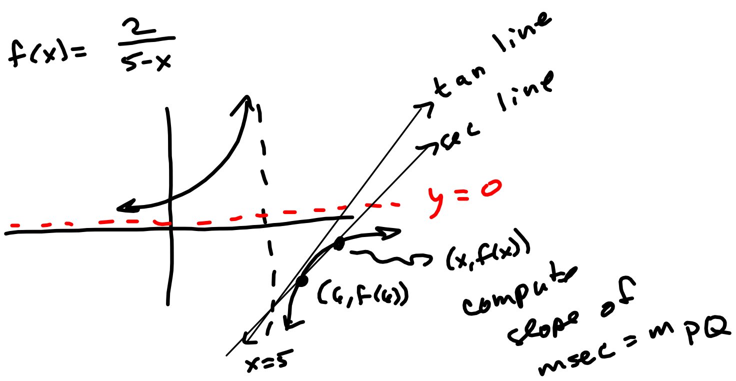
Y2(5.9)
ENTER, 2ND, ENTER

See Spreadsheet! Click Here!

Y2(5.99

This will download the spreadsheet. Go to your Downloads folder and open it up, if you have Excel or Open Office.

ENTER,
2ND, ENTER




```
 Plot1  Plot2  Plot3
\Y1=5/(5-X)
\Y2=(Y1(X)-Y1(6▸
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

```
Y2(5.99
          5.050505051
Y2(5.999
          5.005005005
Y2(5.99999999
                    5
```

Numerical: Calculator or spreadsheet.

$\boxed{E2}$ Let $g(t) = \dfrac{\sqrt{t^2+9} - 3}{t^2}$

Find $\lim_{t \to 0} g(t)$ (if it $\exists$)

$$\frac{x-1}{(x-1)(x+1)} = \frac{1}{x+1} \quad (x \neq 1)$$

$$\& \ \frac{1}{1+1} = \frac{1}{2} = \lim_{x \to 1} f(x)$$

1.6

$$\left(\frac{\sqrt{t^2+9} - 3}{t^2}\right)\left(\frac{\sqrt{t^2+9} + 3}{\sqrt{t^2+9} + 3}\right)$$

$(a-b)\ (a+b)$

"conjugate trick."

$a^2 - b^2$

$$= \frac{t^2+9-9}{t^2(\sqrt{t^2+9}+3)} = \frac{t^2}{t^2(\sqrt{t^2+9}+3)} = \frac{1}{\sqrt{t^2+9}+3} \xrightarrow{t \to 0} \frac{1}{\sqrt{9}+3}$$

$$= \frac{1}{3+3} = \frac{1}{6}$$

Nice way of executing

$$\lim_{t \to 0} \frac{\sqrt{t^2+9} - 3}{t^2}$$

E3 What's $\lim_{x \to 0} \frac{\sin(x)}{x}$?

we'll prove this result in C2.

But don't forget this result!

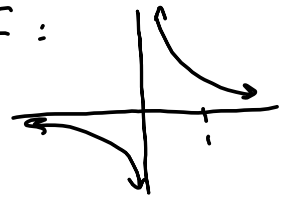
$$\lim_{x \to 0} \frac{\sin(x)}{x} = 1 \; !$$

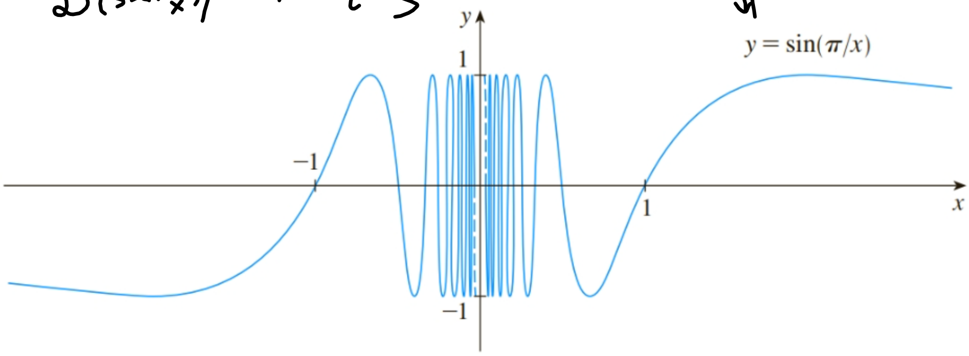
Remember this! We NEED this!

$\boxed{E4}$ $\lim_{x\to 0} \sin\left(\frac{\pi}{x}\right)$ $\nexists$!

$D\left(\sin\left(\frac{\pi}{x}\right)\right) = \mathbb{R}\setminus\{0\}$

$\frac{\pi}{x}$:



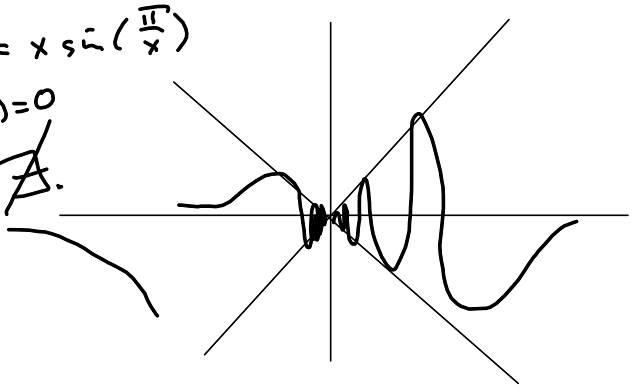


RE 7

$g(x) = x\sin\left(\frac{\pi}{x}\right)$

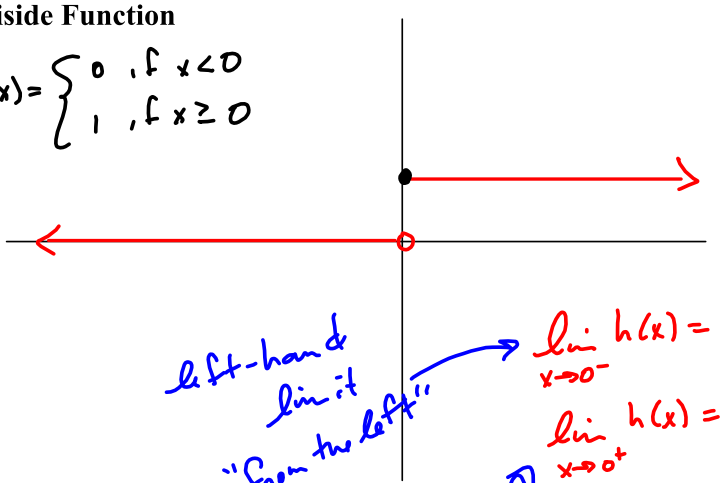
$\lim_{x\to 0} g(x) = 0$

& $g(0)$ $\nexists$.



Heaviside Function

$$h(x) = \begin{cases} 0, & \text{if } x < 0 \\ 1, & \text{if } x \geq 0 \end{cases}$$



Remember:
Dirac Delta Function
δ-function

left-hand limit
"from the left"

$$\left.\begin{array}{l} \lim\limits_{x \to 0^-} h(x) = 0 \\ \lim\limits_{x \to 0^+} h(x) = 1 \end{array}\right\} \lim\limits_{x \to 0} h(x) \nexists$$

Right hand limit
"from the right"

2 **Definition of One-Sided Limits** We write

$$\lim_{x \to a^-} f(x) = L$$

and say the **left-hand limit of $f(x)$ as x approaches a** [or the **limit of $f(x)$ as x approaches a from the left**] is equal to L if we can make the values of $f(x)$ arbitrarily close to L by taking x to be sufficiently close to a with x *less than* a.

Notice that Definition 2 differs from Definition 1 only in that we require x to be less than a. Similarly, if we require that x be greater than a, we get "the **right-hand limit of $f(x)$ as x approaches a** is equal to L" and we write

$$\lim_{x \to a^+} f(x) = L$$

Thus the notation $x \to a^+$ means that we consider only x *greater than* a. These definitions are illustrated in Figure 9.

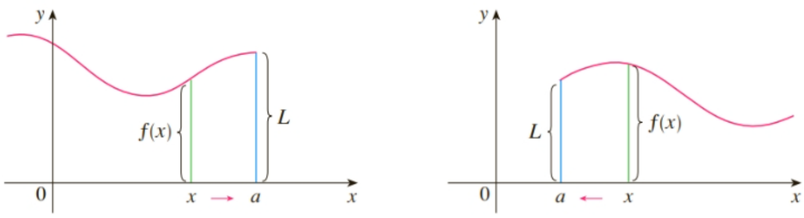


FIGURE 9 (a) $\lim_{x \to a^-} f(x) = L$ (b) $\lim_{x \to a^+} f(x) = L$

x is close to a
|x-a| is small positive number.

3 $\lim_{x \to a} f(x) = L$ if and only if $\left(\lim_{x \to a^-} f(x) = L \text{ and } \lim_{x \to a^+} f(x) = L\right)$

iff
necessary and sufficient.

Limit exists iff the left- and right-hand limits exist and are the same value. Then we can drop the +/- jazz.

Next up:

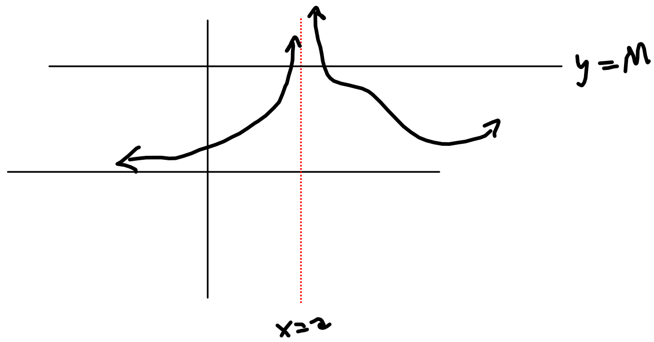
Infinite limits (not to be confused with limits at infinity)

1/x^2

4 Intuitive Definition of an Infinite Limit Let f be a function defined on both sides of a, except possibly at a itself. Then

$$\lim_{x \to a} f(x) = \infty$$

means that the values of $f(x)$ can be made arbitrarily large (as large as we please) by taking x sufficiently close to a, but not equal to a.



It's above M when x is close to 2

Challenge: Give me a number. Make it as big as you want.

Response: I take x close enough to 0 to make f(x) bigger than the number you gave me. In fact, I can do this all day!

CLAIM: $\lim_{x \to 0} \frac{1}{x^2} = \infty$

The skill

Scratch: Want $\frac{1}{x^2} > M$

$\Rightarrow \frac{1}{M} > x^2$

$x^2 < \frac{1}{M}$

$\sqrt{x^2} < \sqrt{\frac{1}{M}}$

$x < \sqrt{\frac{1}{M}}$

Proof: Let $M > 0$

Then $0 < x < \sqrt{\frac{1}{M}} \Rightarrow$

$\sqrt{M} < \frac{1}{x} \Rightarrow$

$M < \frac{1}{x^2}$ ■

$9 < 18$

$\sqrt{9} < \sqrt{18}$

$\boxed{4}$ **Intuitive Definition of an Infinite Limit** Let f be a function defined on both sides of a, except possibly at a itself. Then

$$\lim_{x \to a} f(x) = \infty$$

means that the values of $f(x)$ can be made arbitrarily large (as large as we please) by taking x sufficiently close to a, but not equal to a.

Vertical Asymptotes

6 Definition The vertical line $x = a$ is called a **vertical asymptote** of the curve $y = f(x)$ if at least one of the following statements is true:

$$\lim_{x\to a} f(x) = \infty \qquad \lim_{x\to a^-} f(x) = \infty \qquad \lim_{x\to a^+} f(x) = \infty$$

$$\lim_{x\to a} f(x) = -\infty \qquad \lim_{x\to a^-} f(x) = -\infty \qquad \lim_{x\to a^+} f(x) = -\infty$$

EXAMPLE 9 Find $\displaystyle\lim_{x\to 3^+} \frac{2x}{x-3}$ and $\displaystyle\lim_{x\to 3^-} \frac{2x}{x-3}$. $= -\infty$

$= +\infty$

Domain: $x-3=0$ BAD $\Longrightarrow$ $D = \{x \mid x \neq 3\}$

zeros $2x=0 \Longrightarrow \boxed{x=0}$

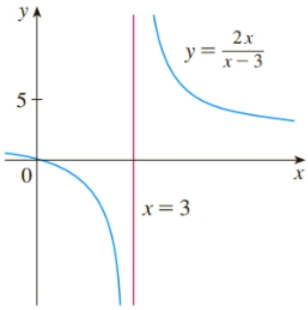
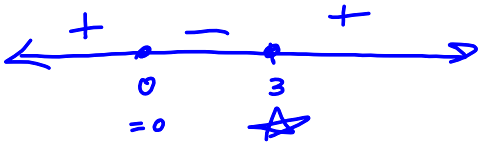


FIGURE 15



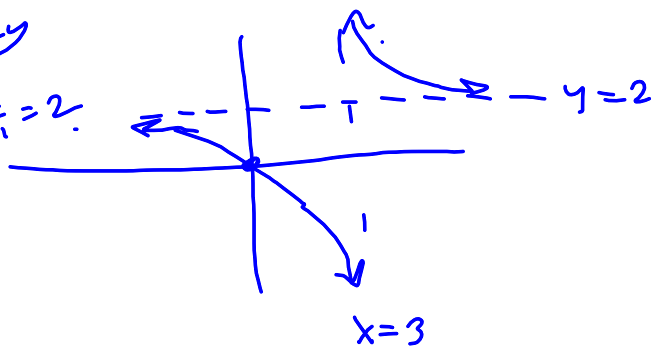
$\frac{2x}{x-3}$

Horizontal asymptote

$\frac{2x}{x-3}$

$\frac{2}{1} = 2 = y$

$\frac{x(2)}{x(1-\frac{3}{x})} = \frac{2}{1-\frac{3}{x}} \xrightarrow{|x|\to\infty} \frac{2}{1} = 2.$



§ 1.6

If f(x) and g(x) agree everywhere, except @ x = a, then

$$\lim_{x\to a} f(x) = \lim_{x\to a} g(x).$$

$$\lim_{x\to 1} \frac{x-1}{x^2-1} = \lim_{x\to 1} \frac{x-1}{(x-1)(x+1)} = \lim_{x\to 1} \frac{1}{x+1} = \frac{1}{1+1} = \frac{1}{2}$$

Need: $\lim_{x\to a} f(x) = f(a)$ if

f is a polynomial, or

f is a rational function & $a \in \mathcal{D}(f)$

f is a trig function & $a \in \mathcal{D}(f)$.

Conjugate Complex #s

$z = a+bi$ has conjugate $z - bi$

$a+\sqrt{b}$ " " $a-\sqrt{b}$ for conjugate trick

$$\frac{3+2i}{1-i} = \frac{(3+2i)}{(1-i)}\frac{(1+i)}{(1+i)} = \frac{3+3i+2i-2}{1^2+1^2} = \frac{1+5i}{2} = \frac{1}{2}+\frac{5}{2}i$$