Hours in varied latitudes

	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
20°	12	12.3	12.9	13	12.8	12.5	12	11.6	11	10.9
30°	12	13.2	13.7	14	13.8	12.8	12	11.2	10.2	10
40°	12	13.5	14.3	14.9	14.2	13.2	12	10.8	9.7	9.1
50°	12	14	15.5	16.1	15.7	14	12	10	8.3	7.9
60°	12	14.9	17.7	18.2	17.8	15	12	9	6.5	5.7

Let t = # of days
Jan 1st ; $t=1$

High Points
$t=172$

J F M A M J
$31+28+31+30+31+21=172$
$59+21=80$
March 21: Spring Equinox

HIGH: 14 hrs in June
Low: 10 hrs in Dec

$a\cos(b(x-c))+d$

$2\cos\left(\frac{2\pi}{365}(t-172)\right)+12$

$2\sin\left(\frac{2\pi}{365}(t-80)\right)+12$

$bx=2\pi$ when $x=365$

$365b=2\pi$

$b=\frac{2\pi}{365}$

We want to find the tangent to the function at the point *P(1, 1)*:

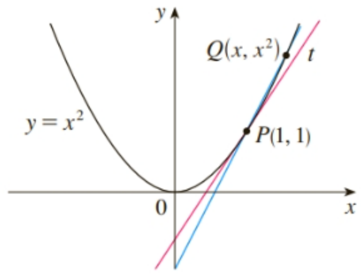


FIGURE 2

$m_{sec} = m_{PQ} =$

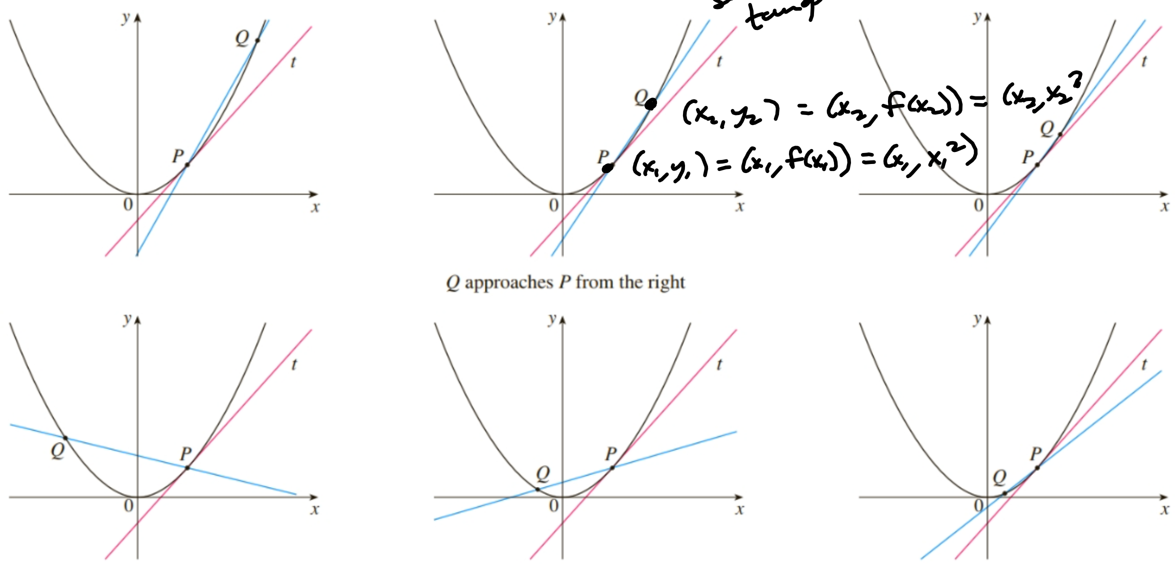
Don't have Excel? Check out Google Docs on your aims.okta.edu. Should be able to save the Excel file to Google Drive, and then open up the spreadsheet from there.

secant

tangent

$(x_2, y_2) = (x_2, f(x_2)) = (x_2, x_2^2)$

$(x_1, y_1) = (x_1, f(x_1)) = (x_1, x_1^2)$



Q approaches *P* from the right

Q approaches *P* from the left

Spreadsheet coming in from the right:

	A	B	C	D	E	F
1	Example 1	P		Q		
2		x1	y1	x2	y2	m(PQ)
3		1	1	2	4	3
4		1	1	1.5	2.25	2.5
5		1	1	1.25	1.5625	2.25
6		1	1	1.1	1.21	2.1
7		1	1	1.01	1.0201	2.01
8		1	1	1.001	1.002	2.001
9		1	1	1.0001	1.0002	2.0001
10		1	1	1.00001	1.00002	2.00001
11						

Just a screenshot of what I did.

2. [–/6.6 Points] DETAILS SCALC9 1.4.003. PRACTICE ANOTH

The point $P(6, -2)$ lies on the curve $y = \frac{2}{5 - x}$.

(a) If Q is the point $\left(x, \frac{2}{5 - x}\right)$, find the slope of the secant line PQ (correct to six decimal places) for the following values of x.

(i) 5.9

m_{PQ} = 2.222222

(ii) 5.99

m_{PQ} = 2.020202

(iii) 5.999

m_{PQ} = 2.002002

See Spreadsheet! Click Here!

This will download the spreadsheet. Go to your Downloads folder and open it up, if you have Excel or Open Office.

1 Intuitive Definition of a Limit Suppose $f(x)$ is defined when x is near the number a. (This means that f is defined on some open interval that contains a, except possibly at a itself.) Then we write

$$\lim_{x \to a} f(x) = L$$

and say "the limit of $f(x)$, as x approaches a, equals L"

if we can make the values of $f(x)$ arbitrarily close to L (as close to L as we like) by restricting x to be sufficiently close to a (on either side of a) but not equal to a.

making the values of x arbitrarily close to a.

Want $|f(x) - L|$ small when $|x - a|$ small

In the sequel: Make $|f(x) - L| < \varepsilon$ when $|x - a| < \delta$

$\lim_{x\to a} f(x) = L = f(a)$

$\lim_{x\to a} f(x) = L$, But $f(a) \neq L$

$\lim_{x\to a} f(x) = L$ But $f(a)$ ∄

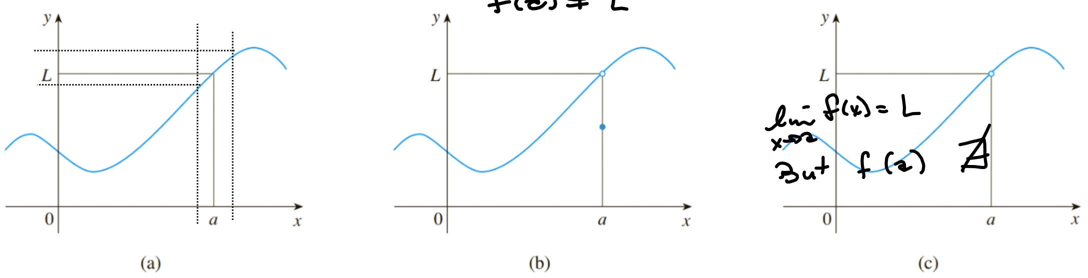
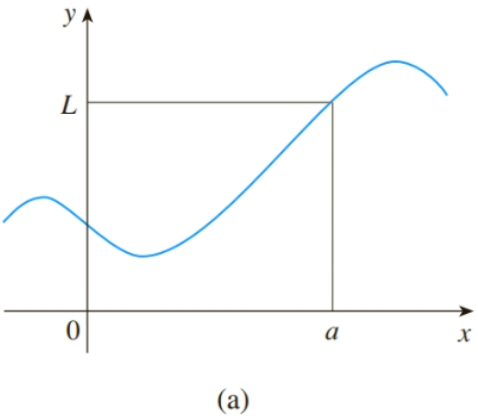
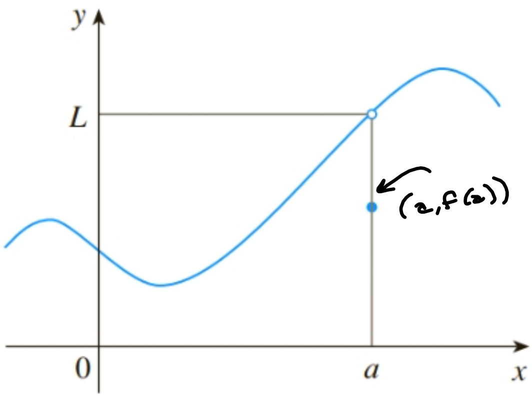


FIGURE 2 $\lim_{x\to a} f(x) = L$ in all three cases

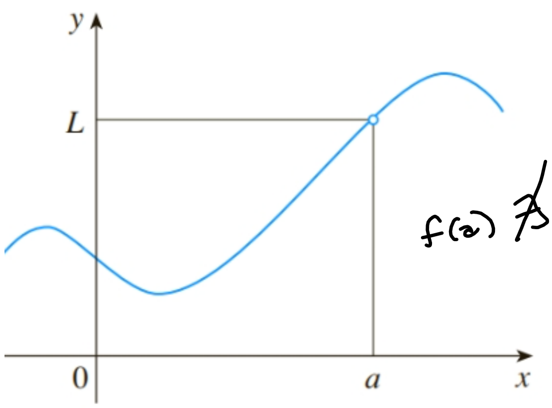


(a)



$(a, f(a))$

(b)



$f(a)$ ∄

(c)

∃ - Exists

∄ - Doesn't exist.

Next up: Use the spreadsheet to do *limits* unerically.

It's nice to have that fall-back, but you want to use either a

$\boxed{E1}$ $f(x) = \dfrac{x-1}{x^2-1}$ $\mathcal{D}(f) = \mathbb{R} \setminus \{-1, 1\} = (-\infty,-1) \cup (-1,1) \cup (1,\infty)$

Find $\lim_{x \to 1} \dfrac{x-1}{x^2-1}$

$x^2 - 1 = (x+1)(x-1) = 0$

$\Longrightarrow x = \pm 1$

Numerical: Calculator or spreadsheet.

$\dfrac{x-1}{(x-1)(x+1)} = \dfrac{1}{x+1}$

$(x \neq 1)$

$\text{\&} \ \dfrac{1}{1+1} = \dfrac{1}{2} = \lim_{x \to 1} f(x)$

$\boxed{E2}$ Let $g(t) = \dfrac{\sqrt{t^2+9} - 3}{t^2}$

Find $\lim_{t \to 0} g(t)$ (if it $\exists$)

$\boxed{E3}$ What's $\lim_{x \to 0} \dfrac{\sin(x)}{x}$?

We'll prove this result in C2.

But don't forget this result!

$$\lim_{x \to 0} \frac{\sin(x)}{x} = 1 \ !$$

$\boxed{E4}$ $\lim_{x \to 0} \sin\left(\frac{\pi}{x}\right)$ $\nexists$!

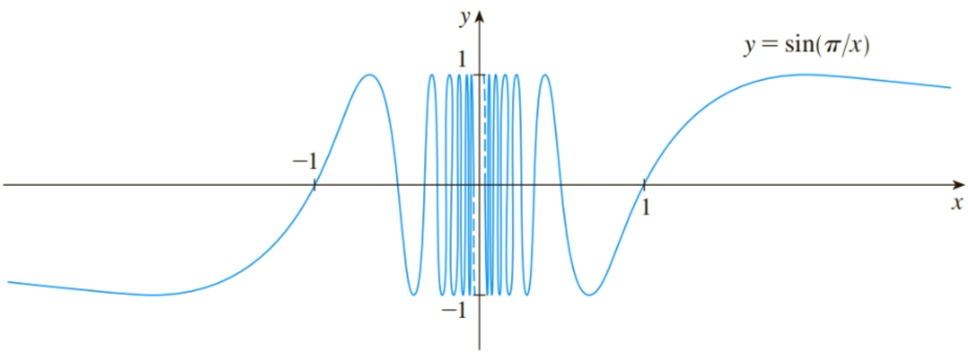


FIGURE 7

Heaviside Function

$\boxed{2}$ **Definition of One-Sided Limits** We write

$$\lim_{x \to a^-} f(x) = L$$

and say the **left-hand limit of $f(x)$ as x approaches a** [or the **limit of $f(x)$ as x approaches a from the left**] is equal to L if we can make the values of $f(x)$ arbitrarily close to L by taking x to be sufficiently close to a with x *less than a*.

Notice that Definition 2 differs from Definition 1 only in that we require x to be less than a. Similarly, if we require that x be greater than a, we get "the **right-hand limit of $f(x)$ as x approaches a** is equal to L" and we write

$$\lim_{x \to a^+} f(x) = L$$

Thus the notation $x \to a^+$ means that we consider only x *greater than a*. These definitions are illustrated in Figure 9.

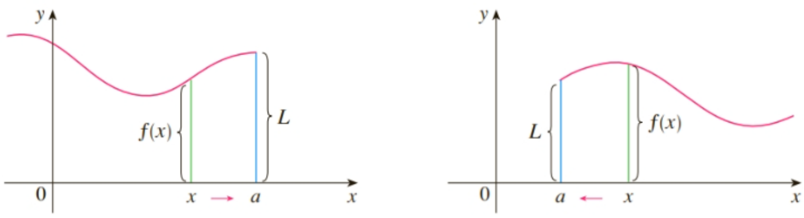


FIGURE 9 (a) $\lim_{x \to a^-} f(x) = L$ (b) $\lim_{x \to a^+} f(x) = L$

x is close to a
|x-a| is small
positive number.

$\boxed{3}$ $\lim_{x \to a} f(x) = L$ if and only if $\lim_{x \to a^-} f(x) = L$ and $\lim_{x \to a^+} f(x) = L$

Limit exists iff the left- and right-hand limits exist and are the same value. Then we can drop the +/- jazz.

Next up:

$\boxed{4}$ Intuitive Definition of an Infinite Limit Let f be a function defined on both sides of a, except possibly at a itself. Then

$$\lim_{x \to a} f(x) = \infty$$

means that the values of $f(x)$ can be made arbitrarily large (as large as we please) by taking x sufficiently close to a, but not equal to a.

$\boxed{4}$ **Intuitive Definition of an Infinite Limit** Let f be a function defined on both sides of a, except possibly at a itself. Then

$$\lim_{x \to a} f(x) = \infty$$

means that the values of $f(x)$ can be made arbitrarily large (as large as we please) by taking x sufficiently close to a, but not equal to a.

Challenge: Give me a number. Make it as big as you want.

Response: I take x close enough to 0 to make f(x) bigger than the number you gave me. In fact, I can do this all day!

Vertical Asymptotes

6 Definition The vertical line $x = a$ is called a **vertical asymptote** of the curve $y = f(x)$ if at least one of the following statements is true:

$$\lim_{x \to a} f(x) = \infty \qquad \lim_{x \to a^-} f(x) = \infty \qquad \lim_{x \to a^+} f(x) = \infty$$

$$\lim_{x \to a} f(x) = -\infty \qquad \lim_{x \to a^-} f(x) = -\infty \qquad \lim_{x \to a^+} f(x) = -\infty$$

EXAMPLE 9 Find $\displaystyle\lim_{x \to 3^+} \frac{2x}{x-3}$ and $\displaystyle\lim_{x \to 3^-} \frac{2x}{x-3}$.

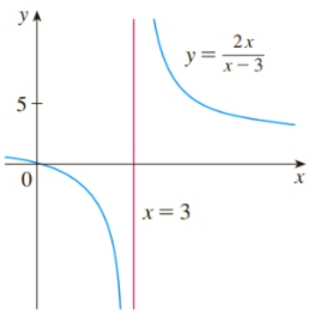


FIGURE 15